MATHEMATICAL MODELING AND CONTINUUM MECHANICS IN THE STEEL INDUSTRY

A.D. Fitt and C.P. Please
Faculty of Mathematical Studies
University of Southampton
Southampton
England SO17 1BJ

telephone: +44 (0) 703 592332 fax: +44 (0) 703 593939
email: cpp@maths.soton.ac.uk

Abstract

The steel industry has provided a rich source of problems that may be studied using mathematical models. In many circumstances the complexity of the physical processes involved means that the formulation of realistic yet tractable mathematical models is a challenge. As well as providing useful predictions, the resulting model equations have frequently raised novel mathematical questions. Below we shall briefly discuss a selection of problems from the steel industry that have been presented at meetings of the European Study Group with Industry, highlighting some of the modeling difficulties and some of the consequent mathematical questions. Two particular problems are discussed in detail. The first relates to the behavior of an anthracite calciner that electrically heats anthracite to high temperatures in order that it may be used later in the steel-making process. A model to explain the observed instabilities in the calciner is presented. The second problem concerns the formation of undesirable oscillation marks on the surface of steel bars during continuous casting.

1. Introduction

The European Study Group with Industry (formerly the Oxford Study Group with Industry) has met annually for twenty six years specifically to consider mathematical models of industrial processes. The steel industry has provided a large number of the many problems that have been considered. In particular, research workers from British Steel (UK) and Elkem a/s (Norway) have regularly attended the meetings. A variety of different areas of steel production have been investigated, ranging from the blasting of iron ore to the rolling of steel plate.

One of the most mathematically productive areas to arise from these meetings has been the study of moving boundary problems. This is an inevitable consequence of the fact that the process of alloy solidification is central to the steel industry. Much of the theoretical work on 'mushy' regions has been directly motivated by problems posed by these solidification processes. Examples of both the analytical properties of these models and the associated numerical methods required to solve them are included in [3], [4], [5], [8], [10] and [11].

Mathematical models involving moving boundary problems may also appear (sometimes quite unexpectedly) in other areas of steel production. Although space does not permit any but the briefest outline of some of these diverse areas, it is instructive to consider them before considering some more detailed examples.

In models of blasting during iron ore mining, the behavior of explosive gases moving through opening cracks must be described. Two sorts of moving boundary are present; the gas front and the crack tip. Typically such problems amount to coupled systems of nonlinear diffusion equations and elliptic elasticity equations where two moving boundaries are to be determined. The structure and numerical details of the relevant solutions have been explored, for example, in [12] and [13].

The Soderberg electrode is a continuously-consumed electrode that is used to distribute the current within an arc furnace. It is composed of a 'paste' that is a mixture of particles of calcined anthracite and pitch, compressed and baked to form a electrically conductive solid. At high temperatures, the paste flows and predictions of the viscosity are required in order that the process may be controlled. In practice, the viscosity is inferred from examination of the slumping of a sample under gravity and applied forces. A discussion of the relevant free surface slow viscous flow problem may be found in [6]. As the paste flows the movement of anthracite particles is size-dependent. Such 'segregation' is undesirable and models to predict the positions of segregation boundaries have been proposed in [1].

The continuous casting process has spawned a vast array of problems. Molten steel is poured at a steady rate into a water-cooled mould. At the bottom of the mould, a bar emerges. This bar consists mainly of molten steel, but has a solid steel skin. Theoretical aspects of the formation of this skin have been discussed in [2]. Eventually, the bar solidifies completely and the steel is ready for further processing. To prevent the steel from sticking to the cooled mould, flux powder is poured onto the molten steel. The flux melts, forming a pool. In order to ensure that the flux lubricates the mould walls, it is necessary to vibrate the mould vertically. A flux layer is formed and this is dragged down by the steel bar being cast, the flux eventually being expelled from the bottom of the mould in solid form. Many models (see, for example, [7] and [14]) have been proposed to determine the rate of flux consumption by considering the position of the interface between the flux and the steel as well as the solid/liquid flux and solid/liquid steel interfaces.

Closely related to the flux consumption problem is the appearance of unwanted 'oscillation marks' on the cast steel bar. These hook and notch shaped marks (which may seriously reduce the commercial value of the final product) arise as a result of a complicated interaction between the mould oscillations, the steadily moving solidifying bar, the flux layer and the molten steel. Predictive models for oscillation marks are discussed in more detail below.

A final example of the occurrence of free boundary problems in the steel industry is furnished by the process of steel bar rolling. In this process, a hot steel ingot is passed (often many times) between rollers to spread the bar into a sheet. Predictions are required (details are given in [15]) of the strain-rate and deformation of the steel in regions near to the rollers, and this may only be accomplished once the position of the steel/roller contact point has been determined.

We now consider two problems in more detail. The first was originally brought to the 1990 Study Group by Dr S.A. Halvorsen of Elkem a/s while the second was presented in several forms at Study Groups and in 1989 by Dr. A. Zoryk of British Steel Technical.

2. The anthracite calciner

In the manufacture of Soderberg electrodes for arc furnaces, carbon is required. This may be produced by heating raw anthracite to a high temperature, thereby converting it to calcined anthracite, a partially graphite material. To accomplish this conversion process, raw anthracite is continuously poured into a cylinder, which is typically 8m long and has a radius of 1m. Inside the cylinder an electric current is passed from a cylindrical carbon electrode immersed in the top of the anthracite to another electrode near the base. The consequent Joule heating burns off any volatile species that are present and then calcines the anthracite. The processed product is slowly scraped from the bottom of the cylinder. Typically, the anthracite takes twenty hours to pass through the cylinder. The internal power dissipation amounts to approximately one megawatt, and temperatures of around two thousand degrees centigrade are regularly achieved. Under normal circumstances, the process works well. Occasionally, however, a local temperature increase may be observed at positions on the cylinder wall. For safety reasons, this is undesirable. An understanding of the calcining process is necessary to anticipate the occurrence of such 'hot spots' and also to predict the degree of anthracite conversion that may be expected in the final product.

Prediction of the performance of calciners constitutes a challenging mathematical problem, since the granular flow of the anthracite, the electrical current flow and the thermal behavior are all closely coupled. While raw anthracite is a very poor electrical conductor, calcined anthracite conducts electricity nearly as well as graphite, suggesting that there is strong interaction between the current distribution and the heating. One of the main points at issue is whether the observed changes in the temperature distribution are due to an inherent instability (possibly resulting from the strong coupling mentioned above), or result from inhomogeneous flow patterns that stem from uneven feeding of the raw anthracite.

A simple model was constructed to investigate the stability properties of the process. In this model, the granular flow was assumed known and only the electrical and thermal properties were to be determined. It was assumed that the electric current was determined by a simple version of Ohm's law with conductivity σ. Heating was produced by ohmic losses, and transferred by a combination of advection and simple linear diffusion. The coupling in the model occurred through the conductivity, which was assumed to depend on the temperature history of the material, and in particular on the maximum temperature encountered. In its simplest version, the model assumed that there was a critical temperature, T_c , at which the transition to calcined anthracite occurred. The electric potential ϕ and the temperature T were determined by the equations

$$\nabla \cdot (\sigma \nabla \phi) = 0$$

$$\rho c_p u \cdot \nabla T = k \nabla^2 T + \sigma \mid \nabla \phi \mid^2$$

where ρ, c_p, k and u denote density, specific heat, thermal conductivity and velocity respectively.

Assuming radial symmetry and uniform flow, the equations predict some interesting behavior. For typical calciners the Peclet number (which compares the inertial time scale to that of conduction) takes a value of approximately 10. This leads inevitably to the existence of temperature boundary layers which in turn affect the flow of current via the conductivity. This boundary layer structure may be analysed and exploited to gain insight into the structure of the solution. One important consequence of the analysis quickly emerges: the heating depends crucially on the ability of the upper electrode to transfer heat vertically to the cold incoming anthracite so that the conductivity increases sufficiently before the material leaves the vicinity of the upper electrode.

The analysis of the problem is closely related to that arising in the study of thermistors [16] and food sterilization [17] where heat and current flow also interact. Novel aspects of the calciner are the preheating by the electrode, the history dependence of the conductivity, and the fact that the calciner is designed so that the cylinder is sufficiently short to prevent the thermal boundary layers from reaching the outer extremities of the cylinder.

3. Oscillation marks in continuous casting

The oscillation mark problem may be approached from a number of different standpoints. An example of an empirically motivated study may be found in [14] while more theoretical approaches are adopted in [7] and [9].

If a predictive model for oscillation mark formation is to be developed, then in general it evidently requires the consideration of heat transfer, solidification, fluid motion, elastic stresses, as well as surface tension and a host of other complicated effects. It is possible, however, to adopt a simpler approach in an attempt to explain the phenomenon. The model presented below should not considered to be complete and is still in need of some development. Nonetheless it provides a tool for examining some of the possible controlling mechanisms.

It is assumed that the flux forms a thin layer separating the mould wall and the steel in which classical lubrication theory is applicable. Adjacent to the mould, the flux moves with the vertically oscillating mould wall. The flux motion adjacent to the steel depends on the properties of the nearby steel. Pressures due to motion in the molten steel are negligible compared to the ferrostatic pressure which must therefore be supported either by the lubricating flux layer or by any solid skin that may form in the steel.

Modeling the behavior of the steel solidification leads to an extremely involved problem, and therefore a simple model was adopted. This is based on the premise that at some point $s(t)$ along the flux layer a solid steel skin forms. Above this point, the ferrostatic pressure is supported entirely by the flux layer; below, the pressure is supported by the steel skin. To completely determine $s(t)$, a thermal problem must be solved and the physical properties of solidifying steel must be accounted for. To avoid these complications, it is assumed that $s(t)$ is prescribed. Below $s(t)$ the solid bar moved steadily downwards, dragging the flux layer with it To determine the flux layer thickness below $s(t)$, it is assumed that the surface of the steel does not deform. The behavior above the point $s(t)$ is less obvious. Two contrasting possibilities have been considered. In the first, the steel cannot support any tangential stress, resulting in the existence of a free surface between the steel and the flux. A second possibility is to assume that the steel has solidified to a sufficient extent, (possibly due to being in a 'mushy' phase) for the flux to be dragged down at the casting velocity of the steel. A further intermediate scenario proposes that the mushy steel can support tensile tangential forces but no compressive forces. In addition to these force balances at the interface the conventional kinematic condition must be imposed to determine the interface motion.

Once the correct model in each region has been identified, boundary conditions must be imposed. Examination of these simple models show that these conditions generate most of the numerical interest and difficulties.

For example, in the case where no tangential stress is permitted between the molten steel and the flux layer, the equations for the flux layer width $h(x,t)$ and pressure p(x,t), with x measuring distance down the flux layer from the molten steel pool are

$$0 \le x \le s(t) \qquad h_t + \left(\frac{h}{2}(V + U\cos\omega t) - \frac{h^2(\rho_s - \rho)g}{12\mu}\right)_x = 0\,, \qquad p = (\rho_s - \rho)gx$$

$$s(t) \le x \le L - U/\omega \sin\omega t \qquad h_t + Vh_x = 0\,, \qquad h_t + \left(\frac{h}{2}(V + U\cos\omega t) - \frac{h^3 p_x}{12\mu}\right)_x = 0$$

Here L is the average length of the flux layer, V denotes the casting speed, while U and ω are respectively the amplitude and frequency of the oscillations. The density and dynamic viscosity of the flux are given by ρ and μ, the steel density is denoted by ρ_s, and g represents the acceleration due to gravity.

The nonlinear hyperbolic equation in the upper region requires a boundary condition at $x = 0$ only when the relevant characteristic enters the solution region. It is argued in [9] that an appropriate condition is to consider that the layer adopts its 'natural thickness' in this case At the bottom of the flux layer the pressure is atmospheric

The boundary conditions at the point $s(t)$ require some care. There are four different cases to consider, depending on the slope of the various characteristics relative to $\dot{s}$. These different cases may be addressed by considering whether or not h and p_x may possess discontinuities.

The complete model may be analysed analytically in some very simple cases, but in general a numerical solution is required. One simple numerical approach is to use the Richtmeyer two-step Lax-Wendroff scheme on a scaled coordinate grid, where the boundaries and position of $s(t)$ have been fixed.

Computations have been carried out for various casting parameters and a number of test cases have been investigated using the no-tangential stress model. Under some circumstances the results are excellent, but in general much modeling and numerical work must be undertaken before the model may be regarded as a useful predictive tool. The difficulties that occur appear to be associated with either the 'natural condition' at the top of the layer, or the structure of the characteristics near to $s(t)$. As described above, the model is mathematically well-posed and understood. As a model for oscillations marks, however, it should be considered to be open-ended and still under development.

4. Concluding remarks

The problems discussed above illustrate the diverse nature of problems that arise from within the steel industry that are amenable to mathematical analysis. The problems posed invite consideration of a vast array of physical phenomena and require drastic simplification in order to render any model mathematically or numerically tractable. Such mathematical models have also raised mathematical questions relating to moving boundaries as well as numerical problems in generating accurate solutions. In all cases the aim is to create insight into the solution of practical problems.

5. References

1. Bergstrom T., Crowley, A.B., Fowler, A.C., Seward, P.E., Segregation of Carbon Paste in a Smelting Electrode, IMA J. Appl. Math., **43**, pp. 83-99, 1989.
2. Blackwell, J.H., Ockendon, J.R., Exact Solution of a Stefan Problem Relevant to Continuous Casting, Intl. J. Heat Mass Transf., **25**, pp. 1059-1060, 1982.
3. Crowley A.B., Ockendon, J.R., Modelling Mushy Regions, Appl. Sci. Res., **44**, pp. 1-7, 1986.
4. Dewynne J.N., Howison, S.D., Ockendon, J.R., Xie, W.Q., Asymptotic Behaviour of Solutions of the Stefan Problem with a Kinetic Condition at the Free Boundary, J. Austral. Math. Soc. B31, pp. 81-96, 1989.
5. Elliot, C.M., Ockendon, J.R., *Weak and Variational Methods for Free and Moving Boundary Problems*, Pitman Research Notes in Mathematics, **59**, 1982.
6. Fitt, A.D., Aitchison, J.M., Determining the Effective Viscosity of a Carbon Paste used for Continuous Electrode Smelting, Fluid Dynamics Research, **11**, pp. 37-59, 1993.
7. Fowkes, N., The Flux of Flux in a Continuous Steel Caster, Research Report, Aug 1989/35, University of Western Australia.
8. Howison, S.D., Similarity solutions to the Stefan Problem and the Binary Alloy Problem, IMA J. Appl. Math., **40**, pp. 147-161, 1988.
9. King, J.R., Lacey, A.A., Please, C.P., Wilmott, P., Zoryk, A., The Formation of Oscillation Marks on Continuously Cast Steel, Math. Engng. Ind., **4**, pp. 91-106, 1993.
10. Lacey, A.A., Tayler, A.B. A Mushy Region in a Stefan Problem, IMA J. Appl. Math., **30**, pp. 303-313, 1983.
11. Ockendon, J.R., Tayler, A.B., A Model for Alloy Solidification in Free Boundary Problems: Application and Theory, **3**, Pitman, 1985.
12. Please, C.P., Wheeler, A.A., Wilmott, P., A Mathematical Model of Cliff Blasting, SIAM J Appl Math, **47**, pp. 117-127, 1987.
13. Paine, A.S., Please, C.P., An Improved Model for the Gas Fracture Process during Rock Blasting, to appear in Int. J. Rock Mech. Min. Sci.
14. Takeuchi, E., Miki, Y., Itoyama, S., Kobayashi, K., Sorimachi, K., Sakuraya, T., Control of Oscillation Mark Formation During Continuous Casting, Steelmaking Conf. Proc., **74**, pp. 73-79, 1991.
15. Tayler, A.B., *Mathematical Models in Applied Mechanics*, Clarendon Press, Oxford, 1986.
16. Fowler, A.C., Frigaard, I., Howison, S.D., Temperature Surges in Current-Limiting Circuit Devices, SIAM J. Appl. Math., **52**, pp. 998-1011, 1992.
17. Please, C.P., Schwendeman, D.W., Hagan, P.S., Ohmic Heating of Foods during Aseptic Processing, to appear in IMA J. Math. Bus. and Ind.