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ACCURATE FLAME SPREAD MODELLING FOR GRANULAR AND STICK PROPELLANTS

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Many codes are now in existence to solve the partial differential equations which model the internal ballistics problem. Most consider mainly granular propellant, differentiating between distinct types by adjustments to the form function and rate of surface regression. This, however, cannot model the differences in the way in which the gas will flow around the propellant, so that for stick propellant where the aspect ratio of a "grain" is very large pipe-flow type effects are ignored and for more closely packed grains the inhibition of the gas from flowing within the propellant bed due to porous medium effects is similarly discounted, leading to inaccuracies in the estimation of flame spreading. A mathematical model is developed to characterize the differences between stick-type and granular-type propellant particles based on a fluid mechanics type consideration of the flow around and within beds of given propellant types. Computations have been carried out using the quasi one-dimensional code ABC1 at Shrivenham in order to highlight the differences between such propellant shapes, and the effects which these can have on flame spreading rates and the generation of pressure waves. The results show that careful fluid mechanical consideration of the flow field around and within propellant beds is vital for accurate prediction of the internal ballistics cycle.

1. INTRODUCTION

In this paper we wish to consider the modelling of flame-spreading in a propellant bed. We assume that the gas/solid flow inside the gun tube may be modelled as a two phase reacting flow, the gas and solid phases interacting during flame spreading and combustion to give rise to heat, momentum and mass transfer. We approach the problem from a quasi one-dimensional viewpoint, with distance x measured along the gun tube from the breech and t representing time. This leads us to partial differential equations for the flow in the two phases which are more manageable than a fully two or three dimensional treatment, but of a higher order of complexity than "lumped" parameter models which can never hope to produce details of the pressure waves present in this type of flow. It should be emphasized that here we are only considering the interaction between the gas and the propellant, and not heat transfer to the tube wall. We mention in passing that in our experience with the code ABC1 for the calculation of quasi one-dimensional unsteady flows, the interphase drag assumes an importance greater than that of a "fine tuning" modelling detail as it can have a crucial effect on whether or not the code exhibits numerical instability. To fix our ideas, we consider specifically the interphase drag $\bar{f}_s$ and heat transfer $\bar{q}_s$ in the gas momentum and energy equations

$$\frac{\partial}{\partial t}(\rho A u) + \frac{\partial}{\partial x}(\rho A u^2) = -A\frac{\partial p}{\partial x} - \bar{f}_s + \dot{m}u_p$$

$$\frac{\partial}{\partial t}\left[\rho A\left(\frac{u^2}{2} + e\right)\right] + \frac{\partial}{\partial x}\left[\rho A u\left(\frac{u^2}{2} + e + \frac{p}{\rho}\right)\right] =$$

$$- p \frac{\partial}{\partial x} (A_p u_p) + m_i e_i$$

$$+ \dot{m}\left(\frac{u_p^2}{2} + e_p\right) - \bar{f}_s u_p - \bar{q}_s$$

Our main aim is to find a way of characterizing the differences between granular (closely-packed propellant particles with an aspect ratio of order unity) and stick propellant where the aspect ratio of the propellant particles is large. In "lumped" parameter models the only independent variable in the equations is time, so that there is little scope for modelling the differences between the flows produced by sticks and granules. All that we can do for these models is to adjust the form function coefficients, burning laws and ballistic sizes accordingly, so that no details of the contrast between the "pipe flow" type effects found in stick propellants and the "porous medium" effects found in bags of grains can be included.

In the quasi one-dimensional case, as well as including the effects of different ballistic size, form functions etc, we can also take advantage of the dependence of the flow quantities on the coordinate x. We begin by considering the model presented in [6]. This makes the following assumptions for the interphase drag and heat transfer:

Heat transfer in the case of granular propellants is assumed to conform to the correlations [5] where the Nusselt number is taken to be

$$Nu_p = 0.4\, Re_p^{2/3}\, Pr^{1/3} \qquad (1.1)$$

so defining a heat transfer coefficient between the solid and gas phases. The Reynolds number is based on the relative velocity of the two phases and the particle diameter, and thermal quantities are evaluated at the "film" temperature

$$T_f = (T + T_p)/2 \qquad (1.2)$$

where T_p is the temperature at the surface of the propellant, which during combustion will be between the ignition and the adiabatic flame temperatures.

The interphase drag for granular propellant is given by a combination of the correlations for porous beds from [4] and [1]. (For details see [6].)

In the case of stick propellant, [6] assumes that the drag is dominated by the boundary layer, using the well-known Karman-Nikuradse correlation

$$Nu_p = 0.023\, Re_p^{0.8} Pr^{0.4} \qquad (1.3)$$

(see, for example [7]) where once again the diameter of the stick propellant is used to characterize the length. The interphase drag may then be deduced from the Nusselt number by means of the Reynolds analogy

$$c_f/2 = St\, Pr^{2/3} \qquad (1.4)$$

(where St is the Stanton number) to give finally for the shear stress

$$\tau_0 = (\mu u_{rel}/D_p) Pr^{-1/3} (0.023\, Re_p^{0.8} Pr^{0.4}).$$

As far as the mass transfer between the two phases is concerned, we will not consider this problem here, assuming that the ignition criterion and the burning laws are fixed.

Although the four laws given above have been used by a wide range of internal ballisticians apart from the author of [6], they are clearly not ideal. In the next section we discuss some of the shortcomings of this approach, with particular reference to the case of stick propellant.

2. SHORTCOMINGS OF THE USUAL MODEL FOR INTERPHASE TRANSFER EFFECTS

In discussing the defects in the model presented above, we must first consider whether it is in any way sensible to model the physical problem as a two-phase reacting flow. It seems apparent that in the case of a charge consisting of a bag of grains, the bag will soon rupture in a real firing and the grains will become distributed throughout sizeable regions of the flow. So probably in this case the assumption that the grains form a second "phase" in the flow is reasonable. In the case of stick propellant, however, it is clear that we are on much shakier ground. Most charges consisting mainly of stick propellant are well packed, and the sticks support each other under loading to produce an overall structure which is much less likely to be dispersed around the flow. It is possible that some sort of treatment of the propellant bundle as an elastic body (see for example [8]) is more correct than a two-phase flow approach, but for the purposes of this paper we will assume that we are justified in writing down two-phase flow equations for the stick propellant case, and attempt to derive the most accurate possible model based on this.

In the case of stick propellant the correlations used are for flow in circular pipes. When a charge is composed of long propellant sticks there will also be flow in the interstices between the sticks, so that some adjustment of the law for stick propellant is needed. In [6] the author remarks that any attempt to adjust the Karman-Nikuradse correlation to take into account the interstitial flow must be regarded as "pure speculation". This may be true, but it is nevertheless the case that the interstitial flow may be very important indeed in the flame spreading process. Experiments with closely packed hexagonal sticks have shown that inadequate flame spread can result when the interstices are not present. Sticks with and without central holes are both in use so it is clear that we must make some statement about the interstitial flow. Note also that slotted tube and non-circular propellant shapes are not taken into consideration in the model as it stands. It may also be the case that under the extreme loading conditions encountered in a real firing the tubes will experience bending, leading to secondary flows superimposed on the basic "pipe" flow. These may have a drastic effect on the heat and momentum transfer characteristics of the flow. (Although of course a quasi one-dimensional model could never hope to predict these.)

Apart from the geometrical inaccuracies mentioned above a much more serious defect of the stick laws is that they are relevant only to a steady fully developed incompressible turbulent attached boundary layer. It is clear that when there are pressure waves present in the flow there will be reverse flow and so the boundary layers will separate in some circumstances. Computations carried out with data from the 105 mm light gun indicate clearly the high velocity gradients and areas of reversed flow present. Quite apart from these difficulties there is the question of compressibility. Pressures of the order of hundreds of megapascals mean that this is a major effect inside a gun tube, and it is unlikely that a purely incompressible analysis will be sufficient. Also the assumption that the boundary layer is a fully developed one needs justification. Clearly there may be some parts of the tubes and interstices where a hydrodynamic and thermal entry length solution would be more appropriate.

The question of boundary layer blow-off must also be considered. In the case of the stick laws, the author in [6] chooses to set the heat transfer between the two phases equal to zero (thus ensuring that the drag is also zero) whenever combustion occurs. The reasoning behind this is that whilst combustion is taking place there is considerable mass transfer between the two phases, and we may now think of the propellant sticks having porous surfaces, with a non-zero normal gas velocity. (Usually termed the "injection" velocity.) It was noted in [3] that when such blowing is of an order of magnitude larger than the vertical velocity of the boundary layer, the boundary layer will be "blown off" and an essentially inviscid rotational layer will be formed near to the wall.

The whole question of how the drag is influenced by combustion does, however, seem to be poorly understood.

Even if we do accept all the shortcomings of the Karman-Nikuradse correlation, we are still left with the problem of how to define the characteristic length and velocity in the Reynolds number. There is some confusion in [6] regarding the length scale, but clearly the tube radius of the cord would seem to be a sensible length scale. With regard to the correct velocity scale, we assume that the flow is close to "plug" flow so that the approximation of [6] of using the relative phase velocity rather than the mean flow velocity is reasonable.

Having highlighted some of the problems in modelling the heat and momentum transfer for a stick propellant, we turn to the case of granular charges. The "patching" of the correlations proposed in [1] and [4] suggests that the given law is capable of simulating a wide range of bed conditions, and indeed agreement of the correlation with experiments has been found to be good in a wide range of cases (see [4]).

For the heat transfer in both types of propellant, we assume that the "Reynolds analogy" will allow the calculation of a heat transfer coefficient from the definition of the drag. To calculate the heat transfer term $\bar{q}_s$ in the gas phase energy equation, some definition of the surface temperature of the propellant will be needed. The method which we use to compute this temperature will also affect the accuracy of our model.

We also note that the expressions used in [6] only estimate the STEADY-STATE interphase drag and heat transfer. We anticipate that the flow will be influenced by a "virtual mass" effect caused by unsteadiness. (See, for example [2] for an explanation of this phenomenon in the case of the unsteady motion of a sphere through fluid.) This additional drag is estimated in [6] by the inclusion of "virtual mass coefficients" in the partial differential equations, which multiply terms in the momentum and energy equations. Although this constitutes a practical solution to the problem, the difficulty of ascribing a value to these cofficients makes its value doubtful.

Having accepted that for granular propellant the popular existing model for heat and momentum transfer is probably sufficiently accurate, we consider what possible improvements may be made to the stick propellant model.

3. IMPROVEMENTS TO THE MODEL FOR STICK PROPELLANT

Before we can consider improving the present model, we must establish some basic facts about the flow inside the sticks. We must consider whether the boundary layer flow is of fully developed or entry length type, and also whether boundary layer blow off is likely to occur so that the drag should be set to zero in the presence of ignition.

In order to determine estimates for the hydrodynamic and thermal entry lengths we consider some typical data, chosen for example for a light gun. Assuming a propellant stick hole radius of 1.2 mm, and basing the Reynolds number on this and the gas phase velocity, the Reynolds number may be calculated using an unsteady quasi one-dimensional code. The resulting mean Reynolds number of order 1,000,000 shows that certainly the flow is, as expected, turbulent. We consider a thermal entrance region where the temperature and velocity profiles develop simultaneously. The results of [10] and [7] suggest that the ratio of the Stanton number for the thermal entry region to that for the fully developed flow is certainly closer to unity than

$$St/St_{fd} = 1 + 1.3/(x/D)$$

For the type of sticks which we are considering the aspect ratio x/D has a maximum value of the order of 500 so that unless there are a large number of separate boundary layer developments within a stick, it seems that we are well justified in neglecting the hydrodynamic and thermal entry lengths.

We may also estimate the likelihood of boundary layer blow-off. Although agreement is not universal on the injection velocities required for lift-off of the boundary layer to form a free shear layer, clearly if

$$U_{inj}/U_{rel} > 0(Re^{-1/2})$$

in the laminar case then we may expect some sort of blow-off to occur. In the case of the turbulent boundary layer, a commonly used criterion (see, for example [3]) is

$$U_{inj}/U_{\infty} > 0.02$$

We may estimate the injection velocity as follows: considering once again the example of a light gun, the rate of surface regression of the propellant stick is given by

$$D_s df/dt = -Bp^{a}$$

where $a = 1$, $B = 0.138$ cm/sec/Mpa, and f is the fraction of the ballistic size remaining. This implies that the inner wall of the stick recedes at a rate of 0.138 m/sec, when the pressure is 200 MPa. (A typical pressure for the flow when combustion is fully established.) Assuming that the evolved gas has a density 1000 times smaller than the solid propellant, we see that if the injected gas is constantly removed by the transverse boundary layer flow then approximately

$$U_{inj} = 138 \text{ m/sec.}$$

Boundary layer blow-off is thus likely to occur if the velocity inside a stick is less than the order of 6900 m/sec, and so for all practical purposes we may assume that the boundary layer will not be strong enough to support the injection effect caused by the evolution of gas, at times when the combustion is fully established. During flame spread, the position is somewhat different. A typical pressure early in the internal ballistic cycle is 10 MPa, and this would lead to an injection velocity of 6.9 m/sec. In this case the boundary layer would be blown off only with a transverse velocity of less than 345 m/sec.

With this knowledge, we may fix our ideas on when we should compute the interphase drag and heat transfer. In the case of stick propellant, we assume that interphase drag takes place during times when ignition has not taken place, or when combustion is present but the boundary layer has not blown off. This latter criterion is especially important since gas velocities are typically large near to a flame front. Heat transfer takes place only when there is no ignition. When the propellant is burning, there will be a flame zone separating the surface of a cord from the evolved hot gas and the assumption that the gas is evolved at the adiabatic flame temperature effectively accounts for any heat transfer which may be taking place.

Prescribing the drag to be zero in the later stages of combustion also allows us to make the assumption that the interstices retain their shape whilst the drag is non-zero. Clearly in the later stages of the cycle the sticks will be blown apart, becoming individual cylinders. We assume that this does not take place until sufficient time has elapsed for the boundary layers to have been blown off.

In the case of granular propellant, we assume that there is interphase drag during the entire internal ballistic cycle, as the drag is now dominated by base drag rather than boundary layer effects. Heat transfer takes place only when the propellant granules are not burning, for the reasons given in the previous paragraph.

4. A MODEL FOR INTERPHASE DRAG AND HEAT TRANSFER IN STICK OR GRANULAR PROPELLANT

We may now present a complete model for the interphase drag and heat transfer. From the burning rate equation we may calculate the speed of the receding burning surface, giving the "injection velocity"

$$U_{inj} = 0.005\ Bp^{a}(\rho_p/\rho) \text{ m/sec} \quad (4.1)$$

where the pressure p is measured in MPa. The contribution to the drag from the holes in the sticks is

$$c_{fT} = 0.046\ Re_T^{-0.2}$$

Here the tube Reynolds number is based on the hole diameter, the relative velocity, and the kinematic viscosity evaluated at the gas temperature. This assumes that the flow is very nearly "plug" flow in the tube, so that the relative gas velocity is close to the mean turbulent flow velocity. It also assumes that Sutherland's law is appropriate for the evaluation of the viscosity as a function of temperature.

For the contribution from the interstitial flow, we assume that the sticks are tightly packed so that each interstice is formed by the contact of three cylinders. In this case we define the "equivalent diameter" as

$$D_E = 4A/P = D_1[2\sqrt{3}-\pi]/\pi \sim 0.1027\, D_1$$

where A is the cross-sectional area of an interstice and P is its perimeter length. Then assuming in addition that the actual interstitial drag c_{fI} and the drag c_{fe} for a duct of circular cross-section of diamater D_E are related by

$$c_{fI}/c_{fe} = K_2$$

gives us an expression for the interstitial drag. With regard to this last assumption, for laminar flow the ratio of the two friction coefficients may be shown to be independent of Reynolds number on theoretical grounds. Experiment has also shown this likely to be true in the turbulent case. We take the value of K_2 to be 0.68 (see [9]) giving for the drag due to the interstices

$$c_{fI} = 0.049\, Re_I^{-0.2}$$

where the interstitial Reynolds number is based on the diameter of a propellant stick. Using the fact that

$$c_f = 2\tau_0/\rho(u - u_p)$$

we may derive an expression for the total drag for stick propellant. (See 4.2.) In doing so the contributions from the holes and the interstices have been scaled by a relevant characteristic length (volume/area ratio).

We must also consider the heat transfer. In the granular case we use the correlation of [5], employing a granular Reynolds number based on a "equivalent particle diameter" $D_p = 6V_p/S_p$. The temperature difference multiplying the heat transfer coefficient is taken to be the difference between the gas and ambient temperatures, thereby assuming that the propellant surface temperature remains close to ambient before ignition. Making a similar assumption in the case of stick propellant, we may determine the heat transfer using a standard Reynolds analogy and the expression for the interphase drag. Finally the complete drag model is

$$\bar{f}_s = \bar{F}\, k_2(1-z\theta_1)^{1/2}/(1-z) \quad \text{(granular)}$$

$$= \bar{F}[0.092 D_0 Re_T^{-0.2}/(D_1^2-D_0^2) + 0.0984 Re_I^{-0.2}/D_1]$$

$$\text{(stick, } U_{inj}/|u-u_p|<0.02 \text{ or } T<T_{ign})$$

$$= 0 \quad \text{(stick, } U_{inj}/|u-u_p|>0.02 \text{ and } T>T_{ign}) \tag{4.2}$$

where

$$\bar{F} = \rho_g A_p (u-u_p)|u-u_p|$$

with corresponding heat transfer

$$\bar{q}_s = \bar{H}[0.4 Re_G^{-1/3}/D_p] \quad \text{(granular, } T<T_{ign})$$

$$= \bar{H}[0.092 D_0 Re_T^{-0.2}/(D_1^2-D_0^2) + 0.0984 Re_I^{-0.2}/D_1]$$

$$\text{(stick, } T<T_{ign})$$

$$= 0 \text{ otherwise} \quad \text{(Granular and stick propellant)}$$

where

$$\bar{H} = (T_g - T_{amb})\rho_g c_{pg}|u-u_p| A_p Pr^{-2/3}$$

Here

$$k_2 = \begin{cases} 5.833 k_1, & \varepsilon < \varepsilon_0 \\ 5.833 k_1 \left[\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{\varepsilon_0}{1-\varepsilon_0}\right)\right]^{0.45} & \varepsilon_0 \le \varepsilon \le \varepsilon_1 \\ k_1 & \varepsilon > \varepsilon_1 \end{cases}$$

with

$$k_1 = \frac{0.3 S_0}{6 V_0}$$

$$\varepsilon_1 = \left[1 + 0.02 \frac{(1-\varepsilon_0)}{\varepsilon_0}\right]^{-1}$$

5. INFLUENCE OF THE MODEL ON NUMERICAL RESULTS

In order to evaluate the effect of the new interphase transfer model on numerical results, test cases were run using the quasi one-dimensional code ABC1 with data for a typical light gun burning stick propellant. As well as the model considered above, calculations were performed with no interphase transfer, and a granular model with a "reduction factor". The latter has been used in the absence of a stick propellant model as a crude approximation with the "drag reduction factor" relating the granular to the stick case typically taken to be 0.1

Figure 1 shows the calculated pressures after 1,2 and 6 milliseconds. For the first two of these times ignition is well established in some parts of the flow but the shot has not moved an appreciable distance. Clearly it is in the early stages that an accurate prediction of the drag will have the most effect. The new model is seen to retain the qualitative effects of the zero drag flow where generally more extreme pressure profiles may be expected, whilst the "reduced" model has the undesirable effect of almost completely smoothing out the flow. At 6 ms where the total mass fraction burnt is 0.829, the pressure is no longer influenced by the details of the drag model and has evolved to a typical approximately linear profile from breech to shotbase.

From this we may conclude that although the modelling of the interphase drag and heat transfer will not have much effect on for example the muzzle velocity of the gun under consideration, an accurate representation of these effects is crucial if we wish to consider processes such as the initiation and evolution of flame spread. Inaccuracies may lead to the suppression of shock waves and discontinuities in the flow, or conversely to the overprediction of peaks and troughs of flow variables. In some cases we have found that an overestimation of the drag effect with the consequential large-scale convection of propellant particles may lead to numerical instability so severe that the code cannot be executed without overflow occurring. It is our view that the new model provides an acceptable compromise between the neglect of drag and use of a modified granular model. It also has the advantage that it has been developed with the basic differences between flow around beds of granular and stick propellant in mind and hence will lead to greater accuracy in the computation of such flows.

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Nomenclature

a	Pressure index in burning law
A	Cross-sectional area
B	Burning rate coefficient
c_f	Dimensionless friction factor
c_{fe}	Equivalent friction factor
c_{fT}	Friction factor for flow in holes
c_{pg}	Specific heat of gas at constant pressure
D	Diameter
D_0	Tube hole diameter
D_1	Propellant stick diameter
D_E	"Equivalent" diameter
D_S	Ballistic Size
e	Chemical energy
ε	Porosity
ε_0	Settling porosity
f	Fraction of ballistic size remaining
$\bar{f}_s$	Interphase drag
K_2	Friction factor coefficient
$\dot{m}$	Mass transfer term
Nu	Nusselt Number = $St\ Pr\ Re$
p	Pressure
P	Perimeter length
Pr	Molecular Prandtl Number
$\bar{q}_s$	Interphase heat transfer
Re	Reynolds Number
Re_g	"Granular" Reynolds Number
Re_T	Reynolds Number based in hole diameter
ρ	Density
St	Stanton Number
St_{fd}	Stanton Number for fully developed flow
t	Time
T	Temperature
T_{amb}	Ambient Temperature
T_f	"Film" Temperature
T_{ign}	Propellant ignition temperature
τ_0	Wall shear stress
θ_1	Form function coefficient
u	Velocity
U_{inj}	Injection velocity
U_{rel}	Relative velocity = $u - u_p$
U_∞	Free stream velocity
x	Horizontal co-ordinate
z	Fraction of propellant volume burnt

Subscripts

i	Pertaining to ignitor
p	Pertaining to the propellant or a particle of the propellant
I	Pertaining to interstices

Figure 1 Numerical Results for 105mm Light Gun