A model of oil burnout from glass fabric

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Abstract

A mathematical model is proposed for the process of the removal (by burning) of oil contained in a glass fibre insulation fabric manufactured in Latvia. The small aspect ratio of the fabric allows simplifications to the modelling which reduce the problem to a single nonlinear ordinary differential equation. When the effects of reflected radiation are also included, the differential equation is supplemented by two integral equations. Predictions of the position of the 'burning zone' accord well with observations made at the factory. The effect of the inclusion of extra heating chambers is also examined, and it is found that the temperature gradient in the fabric may be greatly decreased in this way.

1 Introduction

During the industrial process of manufacturing glass fibre insulation fabric, oil is used to assist in the weaving process. The oil must be removed from the fabric, and to do this a special furnace is used The fabric is passed into the furnace, where the oil is heated to burnout temperature by diesel fuel powered heaters. The complete burnout process takes place on the surface of the moving fabric. The resultant high fabric temperatures are known to influence the intrinsic structure of the material and may cause the tensile strength of the glass fabric to decrease.

For this reason it is necessary to model the processes of fabric heating and oil burnout As well as the models reported below, simultaneous investigations were carried out to determine some of the unknown parameters in the problem. These help to justify the assumptions used in the modelling.

2 The formulation of the heat problem

To propose a simple model of the process we do not consider combustion of the oil as it contributes little to the overall heat balance. In any case, considering only diffusive, convective and radiative effects reveals important aspects of the technological process inside the furnace.

The temperature distribution T in the fabric is described by the basic two-dimensional equation:

$$\rho c_p \frac{\partial T}{\partial t} = \lambda \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) - v \rho c_p \frac{\partial T}{\partial x}, \qquad 0 < x < D,\ 0 < y < \delta, \tag{1}$$

where D is the length of the furnace and δ is the thickness of the fabric It is assumed that the fabric moves along the X–axis with the velocity v; ρ, c_p, λ are the density, heat capacity and heat conductivity of the fabric material.

The initial condition is:

$t = 0: \qquad T = T_0$

The boundary conditions along X–axis are given by

for $x = 0: \qquad T = T_0$,

for $x = D: \qquad \frac{\partial T}{\partial x} = 0$,

For the fabric surface two kinds of boundary conditions are considered:

I A simple Stefan-Boltzmann law

for $y = 0: \qquad -\lambda \frac{\partial T}{\partial y} = \epsilon_L \sigma ({T_N}^4 - T^4) - \alpha (T - T_g)$, (2)

for $y = \delta: \qquad \lambda \frac{\partial T}{\partial y} = \epsilon_L \sigma ({T_k}^4 - T^4) - \alpha (T - T_g)$ (3)

Here $T_N = 850^0\ C$, $T_k = 700^0\ C$ are the temperatures of the heaters at the bottom and the top of the furnace, respectively, $T_g = 720^0\ C$ is the temperature of the gas in the furnace, ϵ_L is the emissivity of the fabric material and α is the coefficient of the convective heat transfer between the fabric and the gas in the furnace, taken to be (see [1])

$$\alpha = Nu\ \lambda_g / D, \quad Nu = 0.044 Re^{0.77} T / T_g$$

The form of the boundary conditions (2) and (3) assumes that the distance between the fabric and the heater is large enough so that the effect of possible reflections of the heat flux from the material surface may be neglected In the specific problem of interest, the distance from the fabric to the bottom heater was given by $a = 0.15$m, whilst the distance to the top heater was $b = 0.2$m. Since the length of the furnace is $D = 1.16$m this suggests that instead of the simple boundary condition (2) allowance should be made for the reflected heat fluxes arising from radiative heat exchanges between the bottom surface of the fabric and the bottom heater: such effects were not considered (and therefore (3) was used) for the top heater since it is further from the fabric and has a lower temperature

II With the reflected heat fluxes we therefore obtain

for $y = 0$: $\qquad -\lambda \dfrac{\partial T}{\partial y} = -\dfrac{\epsilon_L}{1-\epsilon_L}(\sigma T^4 - q_0) - \alpha(T - T_g),$ (4)

for $y = \delta$: condition (3) is used,

where q_0 and q_L, the reflected heat fluxes, are given by (for details see [2])

$$\begin{cases} q_0(x,t) - (1-\epsilon_0)\displaystyle\int_0^D q_L(\xi_1,t)\frac{a^2}{2[(\xi_1-x)^2+a^2]^{3/2}}d\xi_1 = \epsilon_0\sigma T_N^4(x,t) \\ q_L(\xi,t) - (1-\epsilon_L)\displaystyle\int_0^D q_0(x_1,t)\frac{a^2}{2[(x_1-\xi)^2+a^2]^{3/2}}dx_1 = \epsilon_L\sigma T^4(\xi,t) \end{cases} \qquad (5)$$

$0 < x < D,\ 0 < \xi < D,\ 0 < x_1 < D,\ 0 < \xi_1 < D$.

Here ϵ_0 is emissivity of the heater material.

Since the thickness δ of the fabric (typically equal to 0 2 mm) is an order of magnitude less than its other characteristic sizes, we can define an averaging along the $y-$axis as

$$u(x,t) = \frac{1}{\delta}\int_0^\delta T(x,y,t)dy$$

$$\rho c_p \frac{\partial u}{\partial t} = \lambda\frac{\partial^2 u}{\partial x^2} + \frac{1}{\delta}\left(\lambda \left.\frac{\partial T}{\partial y}\right|_{y=\delta} - \lambda \left.\frac{\partial T}{\partial y}\right|_{y=0}\right) - v\rho c_p\frac{\partial u}{\partial x}, \qquad 0 < x < D$$

Assuming that the temperature does not vary in the y-direction and using the boundary conditions (4) and (3) we obtain for the fabric:

$$\rho c_p \frac{\partial u}{\partial t} = \lambda\frac{\partial^2 u}{\partial x^2} - v\rho c_p\frac{\partial u}{\partial x} +$$

$$\frac{1}{\delta}\left(\epsilon_L\sigma(T_k^4 - u^4) - \alpha(u-T_g) + \frac{\epsilon_L}{1-\epsilon_L}(q_0 - \sigma u^4) - \alpha(u - T_g)\right), \quad 0 < x < D. \qquad (6)$$

where

$$\begin{cases} q_0(x,t) - (1-\epsilon_0)\displaystyle\int_0^D q_L(\xi_1,t)\frac{a^2}{2[(\xi_1-x)^2+a^2]^{3/2}}d\xi_1 = \epsilon_0\sigma T_N^4(x,t) \\ q_L(\xi,t) - (1-\epsilon_L)\displaystyle\int_0^D q_0(x_1,t)\frac{a^2}{2[(x_1-\xi)^2+a^2]^{3/2}}dx_1 = \epsilon_L\sigma T^4(\xi,t) \end{cases} \qquad (7)$$

The simple model using conditions (2),(3) for $u(x,t)$ gives the following one-dimensional equation:

$$\rho c_p \frac{\partial u}{\partial t} = \lambda\frac{\partial^2 u}{\partial x^2} - v\rho c_p\frac{\partial u}{\partial x} +$$

$$\frac{1}{\delta}\left(\epsilon_L\sigma(T_k^4 - u^4) - 2\alpha(u-T_g) + \epsilon_L\sigma(T_N^4 - u^4)\right), \quad 0 < x < D \qquad (6')$$

This equation, together with the initial condition $u(x,0) = T_0$, and both the boundary conditions in the X−direction, is solved by a finite difference method A scheme with a uniform mesh $x_i = ih$, $i = \overline{0,N}$ was used, and the calculation was performed until a quasistationary process was established, i.e. $u(x,t) \equiv u(x)$ The system (6),(7) for the reflected heat fluxes is solved at each time-step $t = t_n$ with the additional iterations $u^{(p)}(x_i, t_n)$, $q_0^{(p-1)}$, $q_L^{(p-1)}$, $p = \overline{1,P}$ until convergence. The integrals in (7) were approximated by Simpson's quadrature formula, the quadrature points being assumed coincident with those of the difference scheme The initial estimates for the fluxes were the following:

$$\begin{cases} q_0^0(x_i, t_n) = 0. \\ q_L^0(x_i, t_n) = 0 \end{cases}$$

Once again, the calculation was performed until a quasistationary process was established, i e. $u(x,t) \equiv u(x)$, $q_0(x,t) \equiv q_0(x)$, $q_L(x,t) \equiv q_L(x)$.

3 The results of the calculations

The numerical results for the models described above were compared with experimental observations to allow parameter ranges to be determined for the emissivities ϵ_0 and ϵ_L and the heat transfer coefficient α.

Comparison of results from the two different models also showed that it is essential to take into account the effects of reflected radiation by using (4).

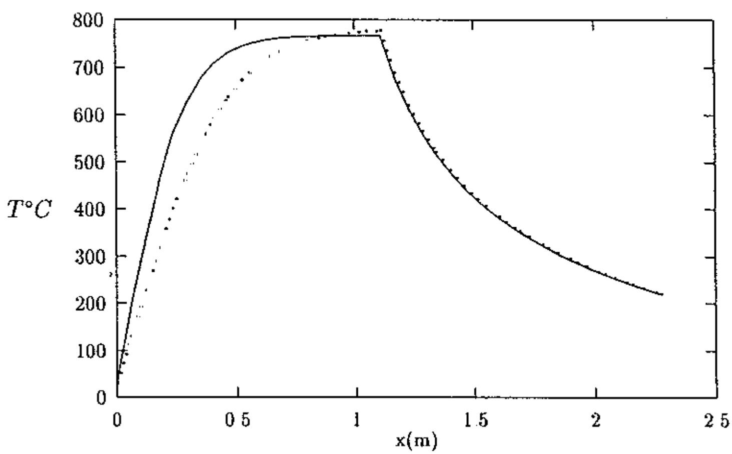

Fig 1 The temperature distribution of the fabric in and outside the furnace for $\epsilon_L = 0\ 6$

....-no reflected radiation (2),(3)

— -reflected radiation included (3)-(5)

In Fig.1 the temperature distribution in the fabric is given as a function of the distance along the x-axis. In this and all other calculations described an initial temperature $T_0 = 30^0\ C$ was used. For distances up to 1.16m the fabric remains in the furnace; after this the fabric has left the furnace and is therefore cooling. The fabric temperature predicted using (3)-(5) (i.e. taking into account reflected radiation) is denoted by a continuous curve, whilst results for the simple Stefan-Boltzmann model ((2) and (3)) are denoted by asterisks. In Fig. 1 a fabric emissivity of $\epsilon_L = 0.6$ was used. As Fig.2 shows, the material temperature depends very strongly on the emissivity of the fabric. That is why a separate experiment was carried out, which showed a very high emissivity of the fabric $\epsilon_L = 0.92$. The point $x = 0.4\ m$ used in the calculations (see Fig.2) was chosen because it corresponded to the actual beginning of the oil burnout. Pronounced burnout in reality occurs within a narrow region, from $x = 0.4\ m$ to $x = 0.5\ m$.

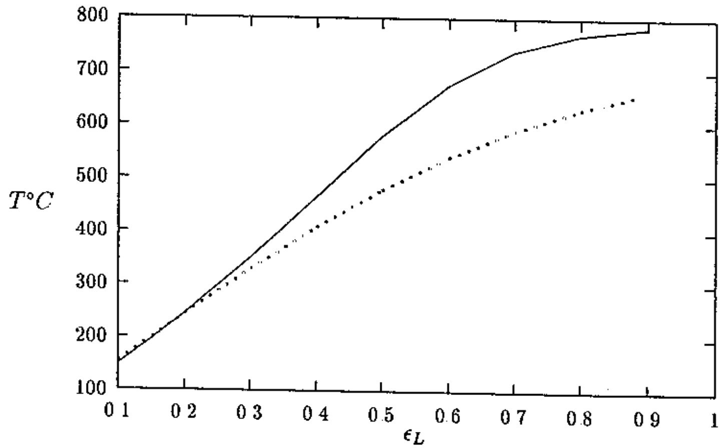

Fig.2 The temperature of the fabric at the point $x = 0.4\ m$ as a function of the emissivity coefficient ϵ_L of the material of the fabric.
. . -boundary conditions (2),(3)
— -boundary conditions (3)-(5)

After the real emissivity of the fabric had been determined the temperature distribution of the fabric in the furnace was checked again, as well as its cooling down after it emerged from there. The results are shown in Fig 3

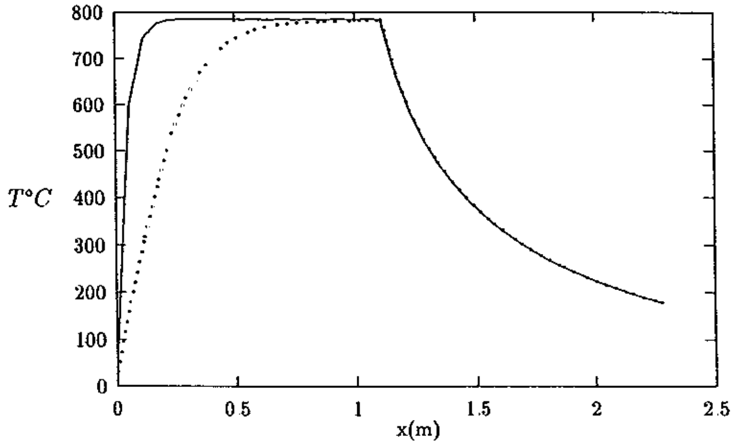

Fig.3 The temperature distribution of the fabric in and outside the furnace for $\epsilon_L = 0.92$.
. . -boundary conditions (2),(3)
— -boundary conditions (3)-(5).

As illustrated there, the difference between the results predicted using the boundary conditions (2),(3) and (3)-(5) became even more important If we take into account that every point of the material is heated not only by the nearest point of the heater, but by all the points of the bottom heater, it means that there is a much faster heating of the material after entering the furnace. It is obvious that the technological process is time-independant· the fabric temperature inside the furnace is constant almost all the time and it is near to 800^0 C

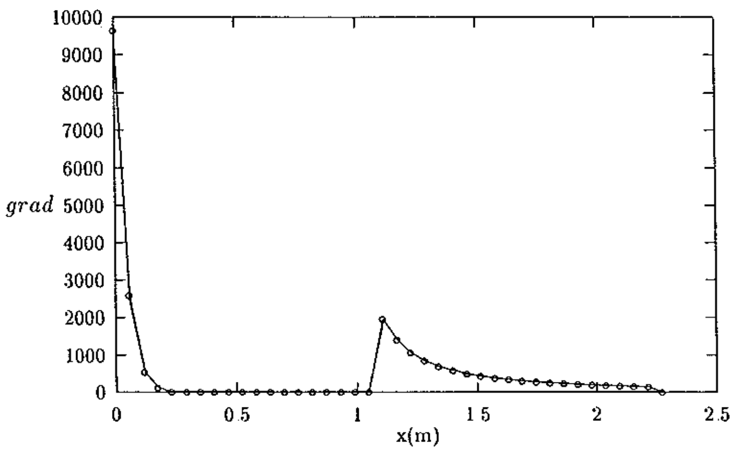

Fig 4 The temperature gradient on the fabric in and outside the furnace

Another important observation is follows: when entering the furnace and after emerging from it the temperature gradients (^{0} C/m) along the fabric are very large (see Fig 4). This causes high thermal stresses in the fabric and may lead to adverse effects in its mechanical characteristics

The results from Fig. 4 reveal that, even when combustion reactions are ignored and a simple heating process is considered, large thermal gradients are present in the fabric It is thought that it may be possible to decrease these by using slow additional pre- and post- heating chambers

Fig.5 shows the temperature distribution along the fabric when such chambers are present (the length of each chamber being 0.6 m). The two curves correspond to two different heat conditions in the chambers. The continuous curve shows the temperature distribution when the upper walls of the extra chambers are unheated and the temperature on the lower walls varies linearly between T_0 and T_N The dotted curve shows results when the upper and lower walls are both heated; on the bottom walls the temperature varies linearly between 250^0 C and T_N, whilst on the top walls of the extra chambers it is held constant at 250^0 C.

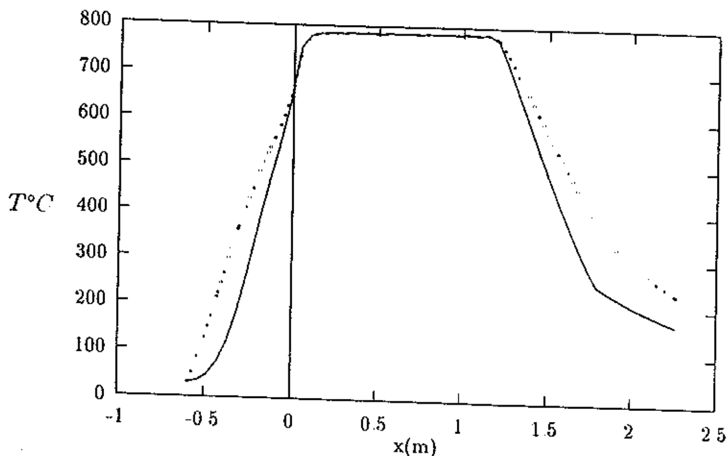

Fig.5 The temperature distribution of the fabric for the case with two additional chambers,
— the chambers without heating,
- the chambers with additional heating

Fig 6 shows the temperature gradient along the X−axis when there are extra heating chambers. It is obvious that having an additional chamber (even if there is no special heater inside it) leads to a decrease in the temperature gradient of approximately five times. In the same manner a strong decrease in the temperature gradient is observed in the slow-cooling chamber. As has already been noted, so far the effects of the burnout process were not included here

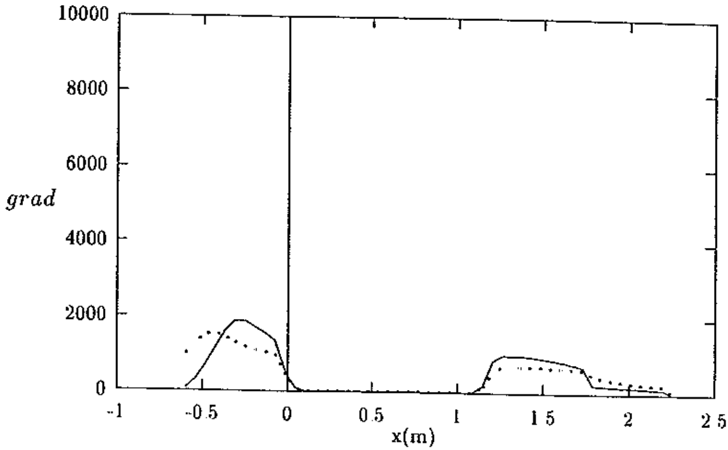

Fig 6 The temperature gradient on the fabric in the furnace.

This process can cause additional temperature gradients inside the furnace, but they seem to be not so large as at the chamber inlet, due to the simple reason that the fabric temperature (about 800^0 C) has already almost reached that of the hottest heater $T_N = 850^0$ C and is higher than the furnace gas temperature $T_g = 720^0$ C

4 Conclusion

It has been shown above how a simple mathematical model may be proposed for the process of oil burnout in glass fabric manufacture. By comparing the results to observations made in the factory, it has become evident that it is necessary to include the effects of reflected radiation if accurate predictions of the burning zone are to be obtained. It is also clear that the mathematical model may be used as a predictive tool to allow the investigation of alternative heating programmes and geometries within the furnace. It is worth pointing out that another key element in the process concerns the fate of the fabric *after* it exits from the furnace. In some circumstances the permanent stress field set up by the cooling of the fabric may adversely affect the quality of the final product. This aspect of the problem is actively being considered at present

References

[1] M A Miheev· Heat transfer Moscow, 1940 (Russian)

[2] R.Siegel/ J.B.Howell: Thermal radiation heat transfer.McGraw/Hill Book Company,1972, Publishing "Mir",Moscow,1975,part 8,pp.294-296 (Russian)