On the Optimum Hand Speed for Two-Blade Razor Shaving

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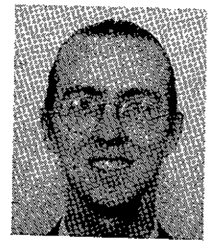

Introduction

THE importance of an efficacious shave has been highlighted by Bloom,[1] who observes

> A man spends 3350 groggy hours (140 days) standing in front of his bathroom mirror during his lifetime

In recent years the single-blade safety razor has been replaced, at least partially, by a heavily advertised two-blade mechanism Evidently, if the public is to be persuaded to purchase a double blade then there must be a perceived and actual tonsorial advantage The key claim of the manufacturers is that whilst the first blade is responsible for a shaving performance hitherto regarded as standard, the cut of the second blade results in a retreat of the now twice-cut bristle to a position below its original surface level, thereby greatly enhancing the effectiveness of the shaving action. Parameters for the success of such a concept are derived and conditions for optimal blade speed are given as a function of blade separation. The details of the model suggested below have deliberately been kept simple, so that the problem as stated and resolved would be suitable for discussion in a first-year undergraduate mathematical modelling course. A further advantage of considering this specific problem is that the implements involved are readily available and experiments are easily performed. Furthermore, any physical parameters which appear in the model may easily be estimated.

A model for two-blade razor shaving

The cut of a single bristle

We begin by examining a phenomenological model for the various stages of a two-blade shave. Fig. 1 shows a general schematic of the passage of the razor across a bed of bristles and a cross-section of the skin taken from Jarrett.[2] The fact that there are many different ways of executing a shave was recognised by Boswell (1791)[3] who relates in his "Life of Johnson,"

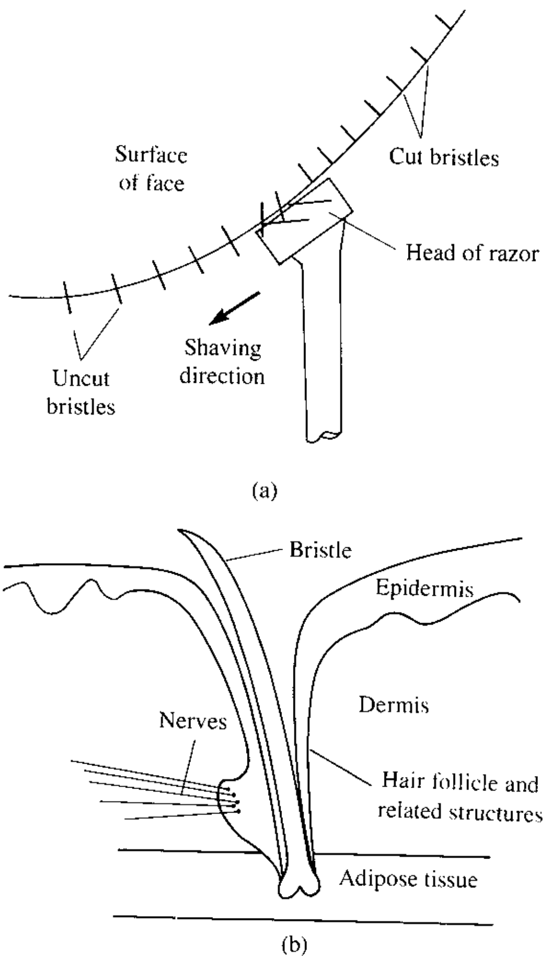

Fig 1 (a). General schematic of passage of razor across face. (b) Cross-section of the skin, taken from Jarrett [2] Not to scale

> ...Talking of shaving the other night in Dr. Taylor's, Dr. Johnson said, "Sir, of a thousand shavers, two do not shave so much alike as not to be distinguished." I thought this not possible, till he specified so many of the varieties in shaving;—holding the razor more or less perpendicular; —drawing long or short strokes;—beginning at the upper part of the face, or the under;—at the right side or the left side. Indeed, when one considers what variety of sounds can be uttered by the windpipe, in the compass of a very small aperture, we may be convinced how many degrees of difference there may be in the application of a razor

Here for simplicity we consider a single bristle which for time $t < 0$ is positioned normal to the skin. The length of bristle below the skin surface is denoted by a and the length standing proud above the skin surface by c_1 (Fig. 2). The shaving action may be broken up into the following stages:

(a) The front blade first encounters the normal bristle, but does not cut it immediately Rather, it drags it in the direction of travel of the blade, pulling the base of the bristle vertically upwards until a critical bristle angle θ_0 is reached, at which time the bristle is cut (Fig. 3). We label the time of this first cut $t = 0$, and measure the x-axis from where the first blade first touched the bristle.

(b) After the first blade has passed, the cut bristle begins to spring back into the skin for times satisfying $0 < t < t_1$; the bristle angle $\theta(t)$ decreases, as does the stubble root height $y(t)$ The length of bristle protruding above the skin surface is denoted by $h(t)$, as shown in Fig. 4

(c) At time $t = t_1$, when the bristle angle is given by $\theta_1 = \theta(t_1)$ the second blade encounters the recently-cut bristle. The final cut takes place, as shown in Fig. 5 (we may assume that the cut takes place immediately; any further "dragging" of the bristle may affect the comfort of the shave but cannot influence its efficiency), and subsequently the bristle settles back into the skin so that $y = 0$

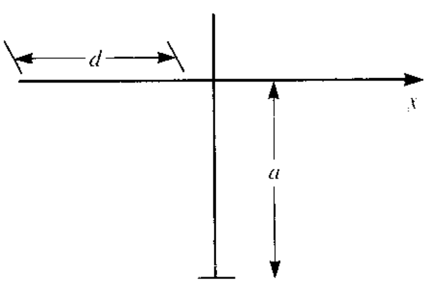

Fig 2 Time $t < 0$: first blade approaches uncut bristle

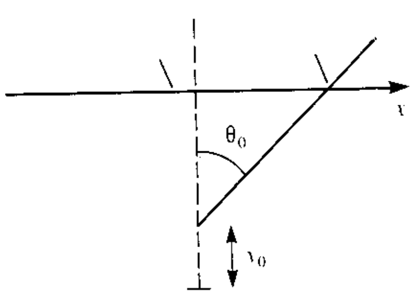

Fig. 3 Time $t = 0$: bristle cut by first blade and released

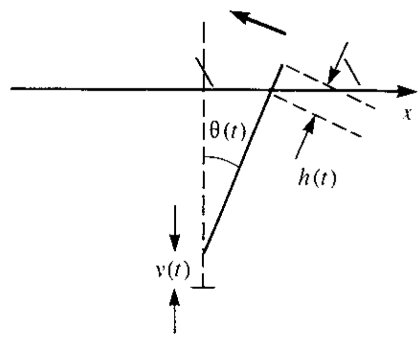

Fig 4. Time $0 < t < t_1$: bristle springs back after first cut

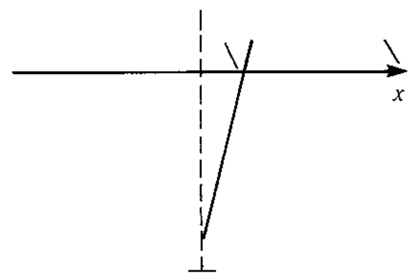

Fig 5. Time $t = t_1$: second cut takes place

Suppose the razor is moved over the skin surface with a velocity V Clearly as $V \to 0$ the bristle has retracted fully before the second blade arrives, whilst as $V \to \infty$ both cuts take place simultaneously This suggests that there is an optimum value of V which will produce the longest-lasting shave. How may this be determined?

Some comments on the key assumptions of the model described above are apposite First, it is clear that the blade does not cut the bristle until it has "dragged" it some distance If the bristle was cut while normal, there would be no point having the second blade and the whole mechanism would fail to function The physical reality of this "dragging" can easily be confirmed by conducting experiments with a paper guillotine and a length of fuse wire Secondly, we assume that the base of the bristle may only move vertically up and down and the bristle itself behaves as a rigid rod The assumption that the bristle is normal to the skin surface initially is a simplifying one; the fact that shaving with or against the grain produces different results is well known to all wet shavers.

Calculation of the optimum speed

Now that the model has been set up, it is a simple matter to derive an equation for the optimum hand speed V. Fig. 3 shows that θ_0 and y_0 are related by

$$\theta_0 = \cos^{-1}\left(1 - \frac{y_0}{a}\right), \qquad (1)$$

whilst from Fig. 4 we find that for $0 < t < t_1$ we have

$$h(t) = a + (y(t) - a)\sec\theta(t) \qquad (2)$$

Examining Fig. 5, we note that at $t = 0$ the second blade was at a position $x = -(d - a\sin\theta_0)$, whilst at $t = t_1$ its position is $x = (a - h(t_1))\sin\theta_1$. Hence,

$$Vt_1 = (a - h(t_1))\sin\theta_1 + d - a\sin\theta_0,$$

so that using (2) we find that t_1 is determined by

$$a\sin\theta_0 + Vt_1 - d = (a - y(t_1))\tan\theta_1. \qquad (3)$$

Our goal is simply to maximize $h(t)$ as a function of V, and to accomplish this we must clearly develop submodels for $y(t)$ and $\theta(t)$. For our purposes, we proceed in an elementary fashion. Since it seems plausible that the skin under some circumstances acts not unlike a linear spring, we assume that the damping exactly balances the restoring force, so that

$$y(t) = y_0 e^{-\alpha t}$$

where y_0 and α are constants to be determined from experiment. Similarly, we assume that

$$\theta(t) = \theta_0 e^{-\beta t},$$

where y_0 and θ_0 are related via (1) and β is to be determined from experiment. We seek a value t^* of t such that

$$\left[\frac{dh}{dt}\right]_{t=t^*} = 0$$

and it is easily confirmed that

$$\frac{dh}{dt} = \frac{dy}{dt}\sec\theta + (y - a)\frac{d\theta}{dt}\tan\theta\sec\theta,$$

so that, since $\sec\theta \neq 0$, we must solve the equation

$$-\alpha y_0 e^{-\alpha t^*} - \beta\theta_0 e^{-\beta t^*}(y_0 e^{-\alpha t^*} - a)\tan(\theta_0 e^{-\beta t^*}) = 0. \qquad (4)$$

The optimum hand speed and corresponding cut length $h(t)$ will then be given by

$$V^* = \frac{d + \dfrac{\alpha y_0}{\beta\theta_0} e^{(\beta-\alpha)t^*} - a\sin\theta_0}{t^*},$$

$$h^* = a + (y_0 e^{-\alpha t^*} - a)\sec(\theta_0 e^{-\beta t^*}).$$

Before considering the implications of these results, let us pause to consider some properties of the constants which have been introduced into the model. Clearly the constants α and β will vary from person to person. They are also crucially dependent on age, as the elasticity of the skin decreases with increasing years. The constant y_0 may be thought of as a measure of "bluntness" and will increase with the age of the razor. Since a blunt razor pulls the bristle farther before cutting, y_0 may also be considered to have dimensions of pain per unit shave.

Results from the model

In order to give quantitative results, some values must be assigned to the constants. Conversations with fellow wet shavers elicited the information that most shaving takes place at a speed which is not below $0.02\,\mathrm{ms}^{-1}$ and is not greater than $0.2\,\mathrm{ms}^{-1}$. To estimate a, we consulted an elementary human biology text. Wood and Bladon[4] estimate the thickness of the epidermis as being between 0.06 and 0.1 mm, whilst the dermis varies in depth from 2 to 4 mm. The layer of adipose tissue in which the base of the bristle is seated has a depth of about 1 mm, so as a first example we assumed that because of the relatively inelastic properties of the skin in the region of the chin the dermis and epidermis would have thicknesses at the lower end of the range and took $a = 2.06$ mm. To determine the blade separation a Gillette "Blue Two" razor was measured, the result being a value for d of 1.5 mm. Values for the "elastic" constants α and β were harder to estimate, but experimentation with a stopwatch in front of a suitable mirror suggested that the timescale of the bristle retraction was of the order of 0.03 s, so an estimate of $\alpha = \beta = 30$ (in SI units) was employed. Finally, a number of somewhat uncomfortable experiments with tweezers suggested that we take $y_0 = 1$ mm, though again this is not an easy parameter to determine and ideally the advice of a dermatologist should be sought.

For these values, $\theta_0 = 1.0303$ and equation (4) becomes

$$1 + 1.0303(e^{-30t^*} - 2.06)\tan(1.0303e^{-30t^*}) = 0,$$

which clearly has a unique solution. This solution may be computed by any standard method (for example, bisection) to be $t^* \sim 1.955 \times 10^{-2}$ s. This gives the optimum shaving speed as $V = 0.036\,\mathrm{ms}^{-1}$, at which value an extra $h^* = 0.27$ mm of bristle is removed solely as the result of the two-blade principle. The rate at which stubble grows varies widely from person to person,† but it seems certain that in some circumstances this could represent the difference between having to shave every other day instead of every day.

Fig. 6 shows plots of h against t and h against V for the conditions described above. One immediate conclusion is that it is better to err on the fast side when one is shaving, as the efficiency drops off rapidly when a hand speed less than the optimum one is employed. One special case should be mentioned, however, which occurs when $d < a \sin\theta_0$, in which case the second blade is already in position as the bristle springs back and the bristle "throws itself upon the sword." Under these circumstances (which are actually unlikely to pertain unless the twin blades are positioned very closely together, in which case the razor is prone to clogging) even a slow hand speed could produce an efficient shave.

Since the determination of some of the important constants requires a little inspired experimentation, it is worth considering how sensitive the model is to changes in the key parameters. When $\alpha = \beta$ (it seems plausible that the two parameters will be of comparable size) it is easy to show that the optimum velocity V^* is directly proportional to α, and h^* is independent of α. So in this case the model is not too sensitive. When α and β are unequal, the model behaves as one would expect; the optimal velocity is affected less than the value of h^*. As an example, we consider the cases where a and y_0 were as given above, and the other parameters were $\alpha = 20, \beta = 30$ and $\alpha = 30, \beta = 20$. This gave values of V^* of $2.4 \times 10^{-2}\,\mathrm{ms}^{-1}$ and $3.2 \times 10^{-2}\,\mathrm{ms}^{-1}$, which corresponded to respective values of h^* of 0.417 mm and 0.102 mm.

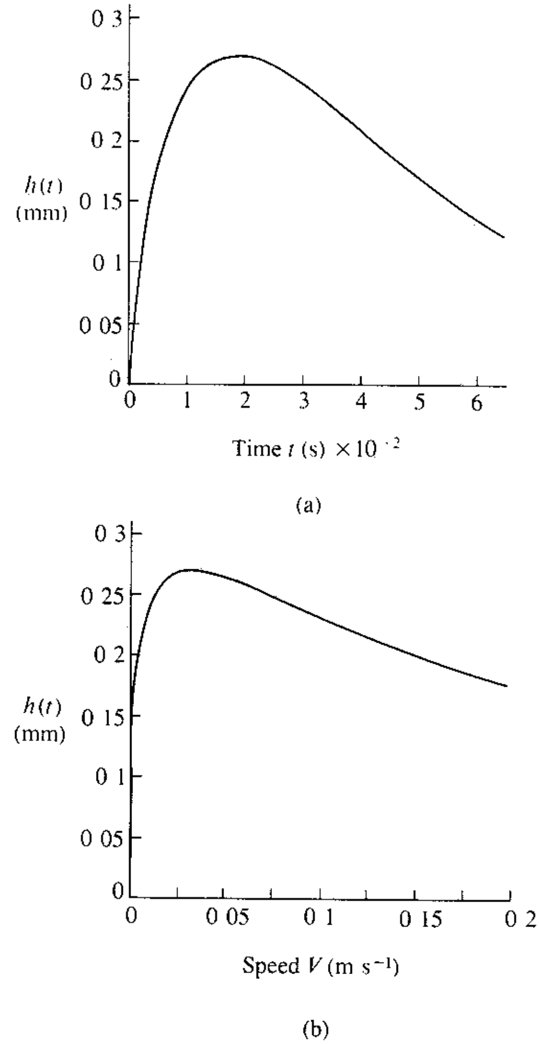

Fig. 6(a) $h(t)$ as a function of time t and (b) $h(t)$ as a function of speed V for the parameters given in the text

Conclusions and reflections

Some comments should be made concerning the applicability of the model to the case of multiple bristles. In many respects the model would require no alteration unless the razor clogged faster and stopped the bristles springing back in the normal manner. To make an accurate model, however, it would be necessary to consider the coupling of the bristles via the skin, which could be treated as an elastic membrane. It is evident that this would greatly increase the complexity of the model.

Clearly the wetness of the skin is crucial to the efficient action of the blade. One interesting case to consider would be a shave performed on completely dry skin, where the "dragging" action of both blades would be at its greatest. Unfortunately we were unable to find volunteers to supply experimental data in this case, and none of the authors felt he was of sufficient moral fibre to provide the evidence himself. Consequently

† Johnson himself conducted frequent experiments, reporting that (Aug. 7th 1779) "*Partem brachii carpo proximam et cutem pectoris circa mamillam dextra rasi, ut notum fieret quanto temporis pili renovarentur.*"

we have assumed above that the skin is always lubricated by at least the critical amount necessary for a reasonably painless shave. In any case, dry shaving is notoriously inefficient, owing to a combination of stick/slip friction effects which lead to "bouncing" of the razor so that the bristle is not cut at skin level, and a natural tendency to perform the shave at an excessively slow speed We leave the last word on the subject to Gale,[5] who enthuses:

> Dr. Norman Orentreich, one of America's most widely quoted dermatologists, believes most men shave poorly because they try to scrape away the beard in a kind of one-step manoeuvre. A good barber, he points out, "lathers, shaves... lathers, shaves. lathers, shaves" . each time the barber is softening the skin making it possible for the blade to glide over the skin. Dr Orentreich continues "I don't care what kind of blade it is—single-edge, double-edge, whatever . But for the cleanest shave and the best-looking skin, I like a razor blade, a sharp knife, cold steel... When a man shaves with lather and a blade razor, his skin looks younger, too."

Acknowledgements

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