# CONTRASTING NUMERICAL METHODS FOR TWO-DIMENSIONAL TWO-PHASE INTERNAL BALLISTICS TEST PROBLEMS

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Motivated by the need for the accurate prediction of ignition and flame spread phenomena, contrasting numerical techniques were used to investigate a prototype two - dimensional, two - phase internal ballistics problem. Good agreement is observed between the two methods, giving confidence that useful two - dimensional predictions of flame spread can be undertaken.

## 1. INTRODUCTION

In recent years a number of quasi one - dimensional two phase flow internal ballistics codes have been developed. These give good agreement with experimental results for such quantities as muzzle velocity and peak chamber pressure. However, for an examination of flame spread phenomena and investigations of igniter performance these codes give insufficient detail, and the early part of the cycle can crucially affect the peak pressure developed. This is particularly important for the understanding of weapon or igniter system malfunctions. We have therefore undertaken the development of a two - dimensional axisymmetric two phase flow internal ballistics code. A similar approach has also been taken in the U.S (TDNOVA[1]) and in Germany (AMI[2]).

Two - dimensional flow calculations should yield much improved modelling of the initial flame spread, and, after further examination of ignition criteria, should become a powerful aid to igniter design. More accurate pictures of the flow near shot base will yield additional information relevant to the perennial problems of heat transfer, wear and erosion

The emphasis, in the early part of this work, has been on ensuring that the model equations are solved accurately, and therefore a number of current numerical methods have been examined The rationale for this step is that it is important to distinguish between results which fail to match experiment because of some shortcoming of the model equations, and inaccuracies which

which result from the numerical solution scheme. We have therefore used two very different techniques, which are described below, and compared results for a number of test problems.

The two numerical methods for which results are shown are based on contrasting approaches. The first is a MacCormack forward/backward finite difference scheme, with an additional diffusion term of the type suggested by Rusanov[3], differenced as in the viscous form of MacCormack's scheme. The additional term is used to suppress the sensitivity of the basic MacCormack scheme to the flow direction, and is required as the flow reverses after initial reflections. This approach has the advantage of largely removing the oscillations near discontinuities in the flow which which are typical of second - order methods, without undue smearing. In addition to this, the method is simple to code and computationally cheap to run.

The second method used is the Weighted Average Flux ('WAF') method due to Toro[4], and is based on the solution of a set of Riemann problems. A number of schemes of this type have previously been formulated (see, for example Godunov[5], Chorin[6], and Roe[7]) and it is the choice of the form used for the flux term which distinguishes them. These schemes are capable of high resolution and accurate capture of shocks and contact discontinuities. the updating flux at any time in the present method is a weighted average of the flux vector across the whole wave structure derived from the solution of the Riemann problem at that time. This is combined with a flux limiter to give a scheme which is oscillation free in the neighbourhood of discontinuities. The result is a simple but robust scheme which may be applied to the solution of systems such as that discussed below.

## 2. MATHEMATICAL MODEL AND NUMERICAL SOLUTION TECHNIQUES

The mathematical model is based on the usual assumptions, that each phase may be represented as a continuum on a scale which is large compared with the particle size. It is assumed that the particles are incompressible and interact with one another through intergranular stress. Interaction between the phases is principally through the interphase drag, and the combustion of the solid particles It is also assumed that the pressure is the same in both phases.

The governing equations are then

$$(\alpha_1\rho_1)_t + \frac{1}{r}(r\alpha_1\rho_1 v_1)_r + (\alpha_1\rho_1 u_1)_x = \dot{m} + m_p$$

$$(\alpha_2\rho_2)_t + \frac{1}{r}(r\alpha_2\rho_2 v_2)_r + (\alpha_2\rho_2 v_2)_x = -\dot{m}$$

$$(\alpha_1\rho_1 u_1)_t + \frac{1}{r}(r\alpha_1\rho_1 u_1 v_1)_r + (\alpha_1\rho_1 u_1^2)_x + \alpha_1 p_x = \dot{m}u_2 - f_s.e_x$$

$$(\alpha_2\rho_2 u_2)_t + \frac{1}{r}(r\alpha_2\rho_2 u_2 v_2)_r + (\alpha_2\rho_2 u_2^2)_x + \alpha_2 p_x = -\dot{m}u_2 + f_s.e_x + \frac{1}{r}(r\sigma_{xr})_r + \sigma_{xx})_x$$

$$(\alpha_1\rho_1 v_1)_t + \frac{1}{r}(r\alpha_1\rho_1 v_1^2)_r + (\alpha_1\rho_1 u_1 v_1)_x + \alpha_1 p_r = \dot{m}v_2 - f_s.e_r$$

$$(\alpha_2\rho_2 v_2)_t + \frac{1}{r}(r\alpha_2\rho_2 v_2^2)_r + (\alpha_2\rho_2 u_2 v_2)_x + \alpha_2 p_r = -\dot{m}v_2 + f_s.e_r + \frac{1}{r}(r\sigma_{rr})_r + (\sigma_{rx})_x$$

$$(\alpha_1\rho_1(\frac{1}{2}(u_1^2+v_1^2)+e_1))_t + \frac{1}{r}(r\alpha_1\rho_1 v_1(\frac{1}{2}(u_1^2+v_1^2)+e_1+p/\rho_1))_r + (\alpha_1\rho_1 u_1(\frac{1}{2}(u_1^2+v_1^2)+e_1+p/\rho_1))_x =$$

$$-\frac{p}{r}(r\alpha_2 v_2)_r - p(\alpha_2 u_2)_x + m_p e_p - (u_2 e_x + v_2 e_r).f_s + \dot{m}(\frac{1}{2}(u_2^2+v_2^2)+e_2) - q_s - Q$$

$$(N_2)_t + \frac{1}{r}(rN_2 v_2)_r + (N_2 u_2)_x = 0$$

where the subscript 1 refers to the gas phase, 2 to the solid phase, and p to the primer Also here $\sigma$ = intergranular stress, $f_s$ = interphase drag, $q$ = interphase heat transfer, $Q$ = heat loss to the walls, these quantities being specified by independent constitutive laws, and $\dot{m}$ = mass addition by burning, $N_2$ = number density of particles per unit area and $\alpha_1 + \alpha_2 = 1$. We also employ the co-volume equation of state for the gas phase, and use the propellant burning law

$$\frac{dr}{dt} = -\beta p^{\alpha}$$

For the present computations, the bed was not loaded and the effects of interphase drag and heat transfer were ignored in order to simplify the comparisons.

It should be mentioned in passing that there are some theoretical difficulties with the present equations as they do not constitute a strictly hyperbolic system. Modelling work is currently in progress on this topic however, and we do not discuss it further here.

## 3. TEST PROBLEMS, RESULTS AND DISCUSSION

The two numerical methods described above have been compared on a number of gas dynamics test cases, one dimensional problems for which analytic solutions are available, as well as on the internal ballistics problems described here. Results were computed for Sod's shock tube problem[8] and for the colliding shock test problem of Woodward & Colella[9]. The WAF method gave better definition with a complete absence of oscillations, but the results of the Rusanov - MacCormack scheme were surprisingly good for such a simple approach. Unfortunately space does not permit us to show these results here.

The internal ballistics test problem considered here is a two - dimensional cartesian analogue of the AGARD test problem posed in 1982[10]. A stick igniter of uniform strength vents into a chamber of square cross - section, with equal volume to that specified for the AGARD test problem, as indicated in figure (1). The propellant is that of the AGARD problem, a 7-hole multitube. For all of the computations described herein, a mesh size of $50 \times 15$ cells was employed. The size of the time step was determined as usual by the satisfaction of a local Courant condition. Figure (2) shows isoparametric plots of pressure and temperature at 0.1ms after the start of primer venting, as the pressure wave reflects from the side wall of the chamber. Figure (3) shows pressure, temperature and both gas velocity components on a plane just off the axis of symmetry at the same time. In figure (4), the pressure, temperature and gas velocities are shown for the plane $x = 0.0608$ metres. It can be seen that in general, the qualitative agreement is very good. Indeed, the isoparametric plots are virtually indistinguishable. Figure (3) suggests that there are some very small differences in the manner in which the venting gases are treated and therefore the amplitudes of the profiles. However, the locations of the shock fronts are almost identical. It should be noted that the difference in the predicted $v$-velocities is due to the fact that in the current implementations of the methods, one has nodes placed at the edges of the computational cells, and the other has cell - centred nodes. For most of the results this only makes a negligible difference, but near to the axis the effect is more pronounced. In figure (4), The zero $u$ - velocity in both cases is indicative of the fact that the disturbance has not yet reached this point of the flow. Clearly, at this early time in the flow, both numerical methods are performing well.

Further results are shown for the later time of 0.75 ms, just after reflection of the initial disturbance from the shot base. Figure (5) shows isoparametric plots of pressure and temperature for both methods, whilst figure (6) contains the temperature, pressure and both gas velocity components on a plane just off the axis of symmetry. These variables are shown again in figure (7), but on the plane $x = 0.0608$ metres. In figure (5), the results from the MacCormack method show a typical small second - order oscillation which results from the shock reflection. The WAF method resolves the reflected shock more accurately, which is to be expected as it uses the local solution of the Riemann problem. Both agree however on the position of the wave front. There is clearly a region near to the side boundary where there is some disagreement between the temperature profiles. The effect is local however, and may indicate that some revision of boundary condition methodology is required. Figure (6) shows more clearly the post - reflection oscillation in the gas pressure produced by the MacCormack method. Again, nevertheless in general agreement is very good, allowing for the differences in mesh - point location described above. In figure (7) the difference in gas temperature at the wall is again visible. The vertical scale on the gas pressure comparison should be noted, as it confirms the 5% disagreement which was observed for the earlier figures.

## 4. CONCLUSIONS

Computations have been undertaken for a problem which is unsteady, two - dimensional and involves strong source terms. In spite of these difficulties, good agreement has been obtained using two very different numerical methods. This gives us confidence that detailed calculations relating to igniter design and to heat transfer characteristics of weapon systems can be meaningfully undertaken.

## ACKNOWLEDGEMENTS

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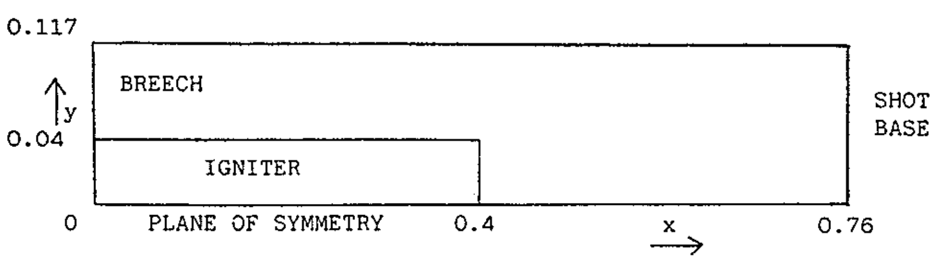



Figure (1) : Schematic showing igniter and chamber geometry (dimensions in metres).

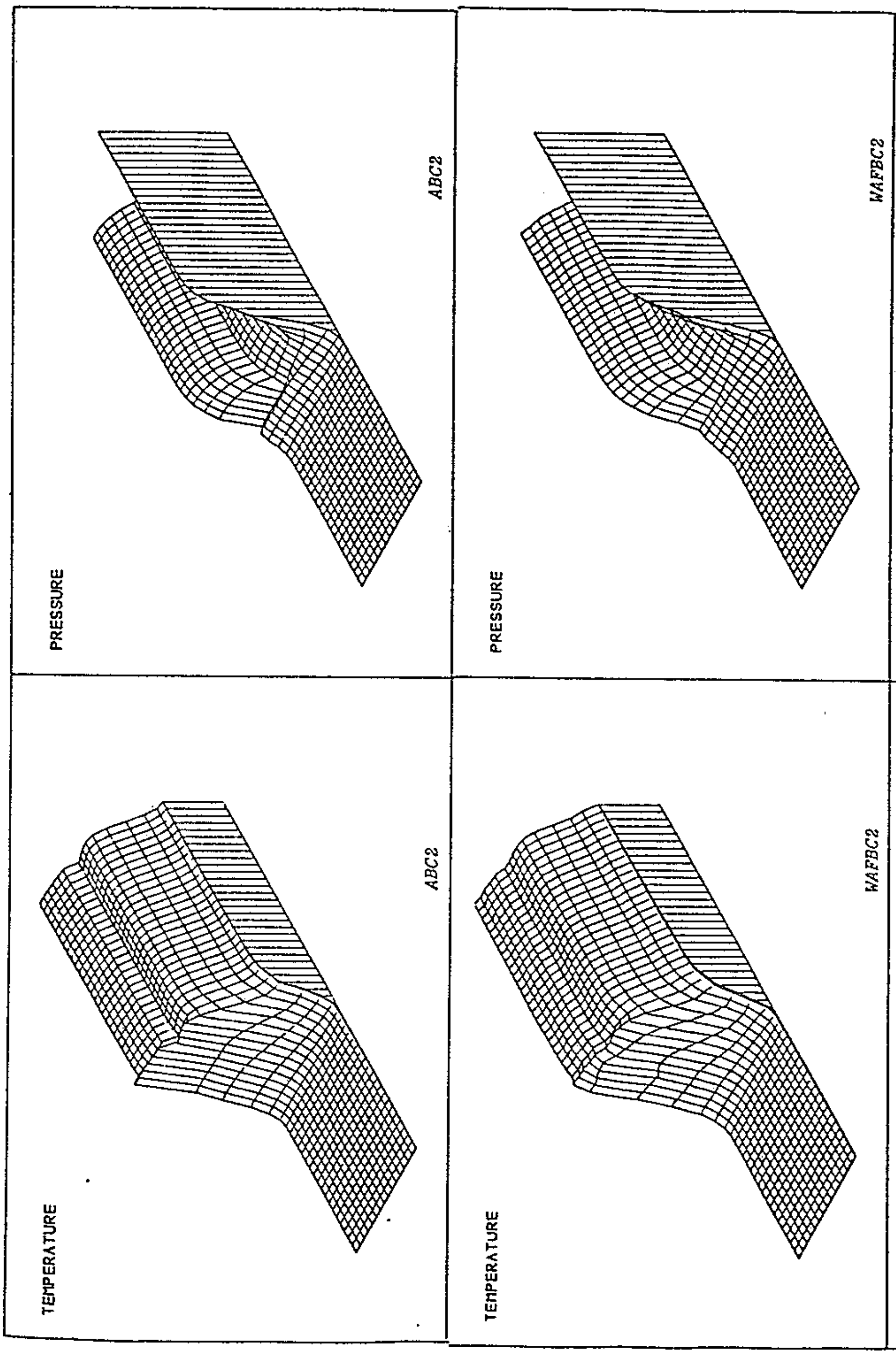
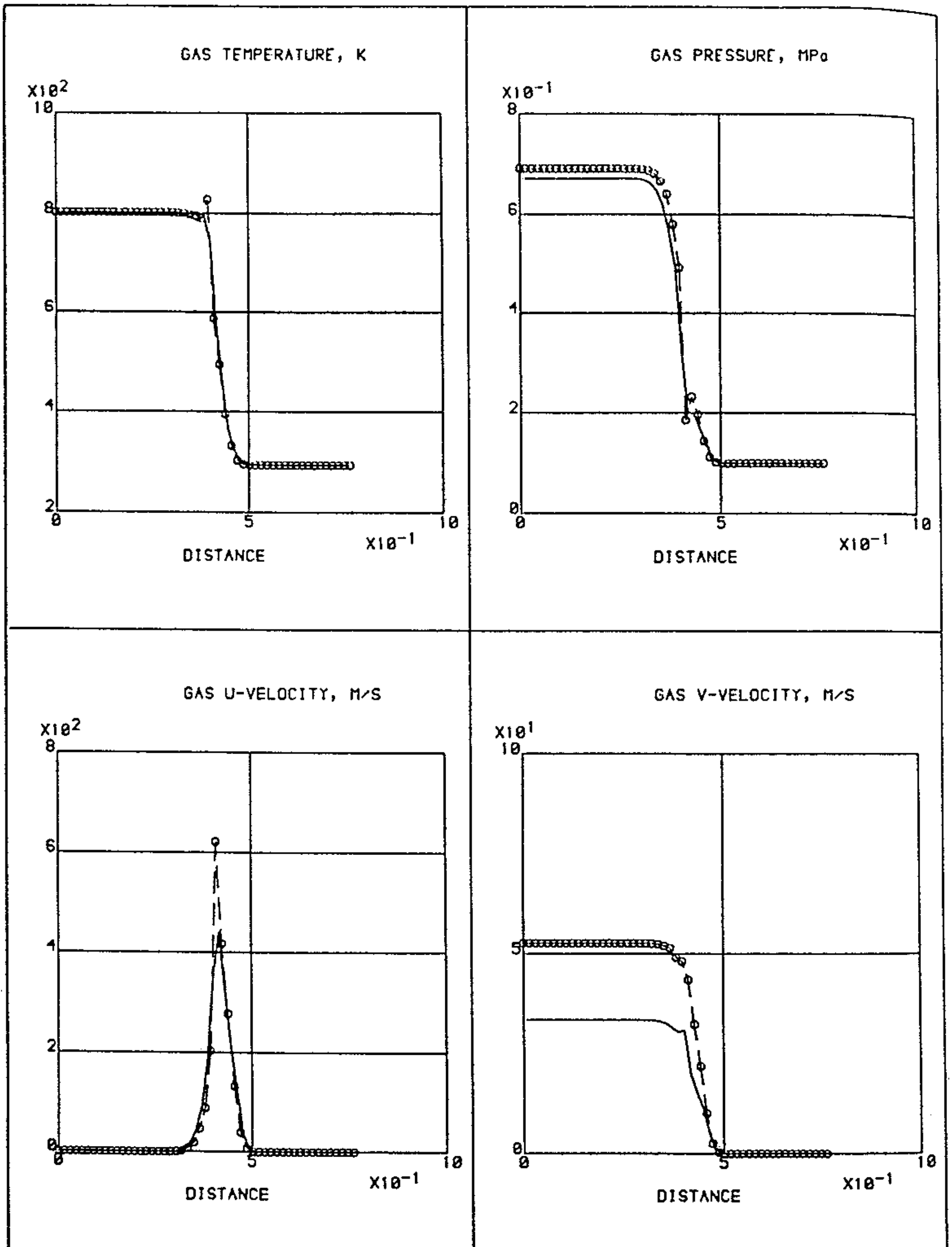
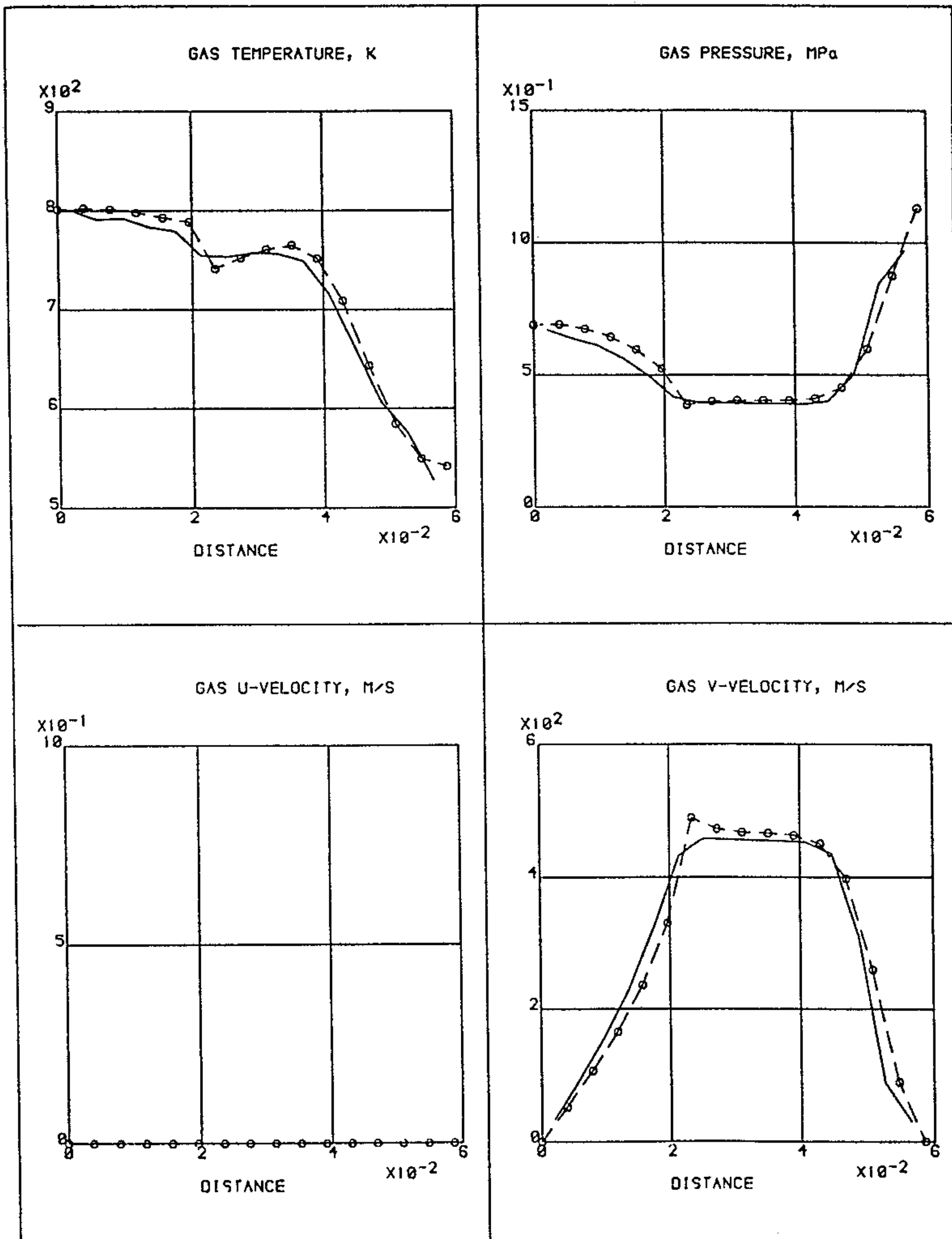



*FIGURE 2 : TWO PHASE 2-D TEST PROBLEM. SOLUTION AT TIME = 0.1 MS.*




TWO-PHASE 2-D TEST PROBLEM. SOLUTION AT TIME = 0.10 MS.
FIGURE 3 :NUMERICAL SOLUTIONS USING WAFBC2 (FULL LINE) AND ABC2 (SYMBOLS PLUS DASHED LINE) FOR J=1.




TWO-PHASE 2-D TEST PROBLEM. SOLUTION AT TIME = 0.10 MS.
FIGURE 4 :NUMERICAL SOLUTIONS USING WAFBC2 (FULL LINE) AND ABC2 (SYMBOLS PLUS DASHED LINE) FOR I=4.

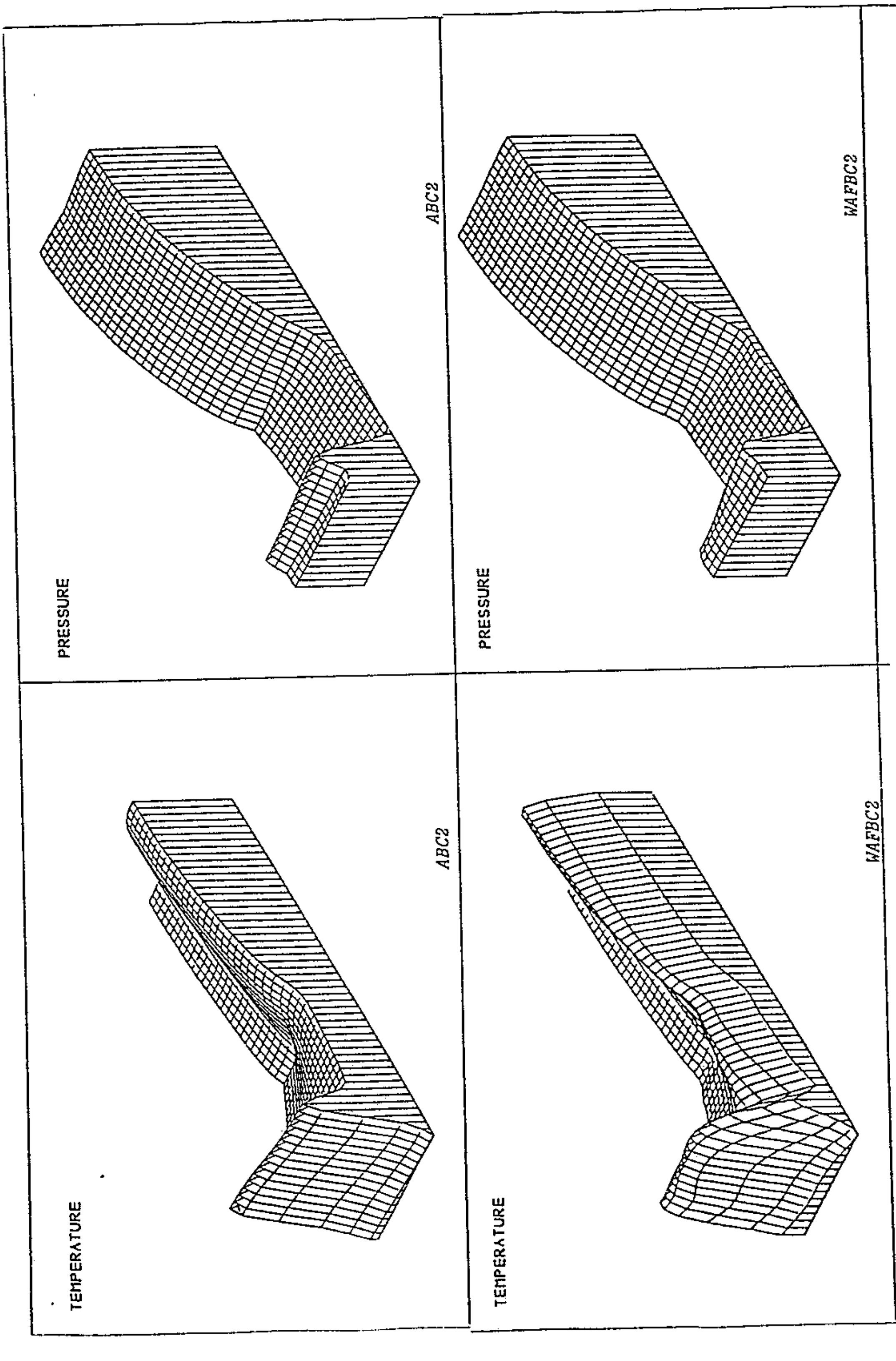
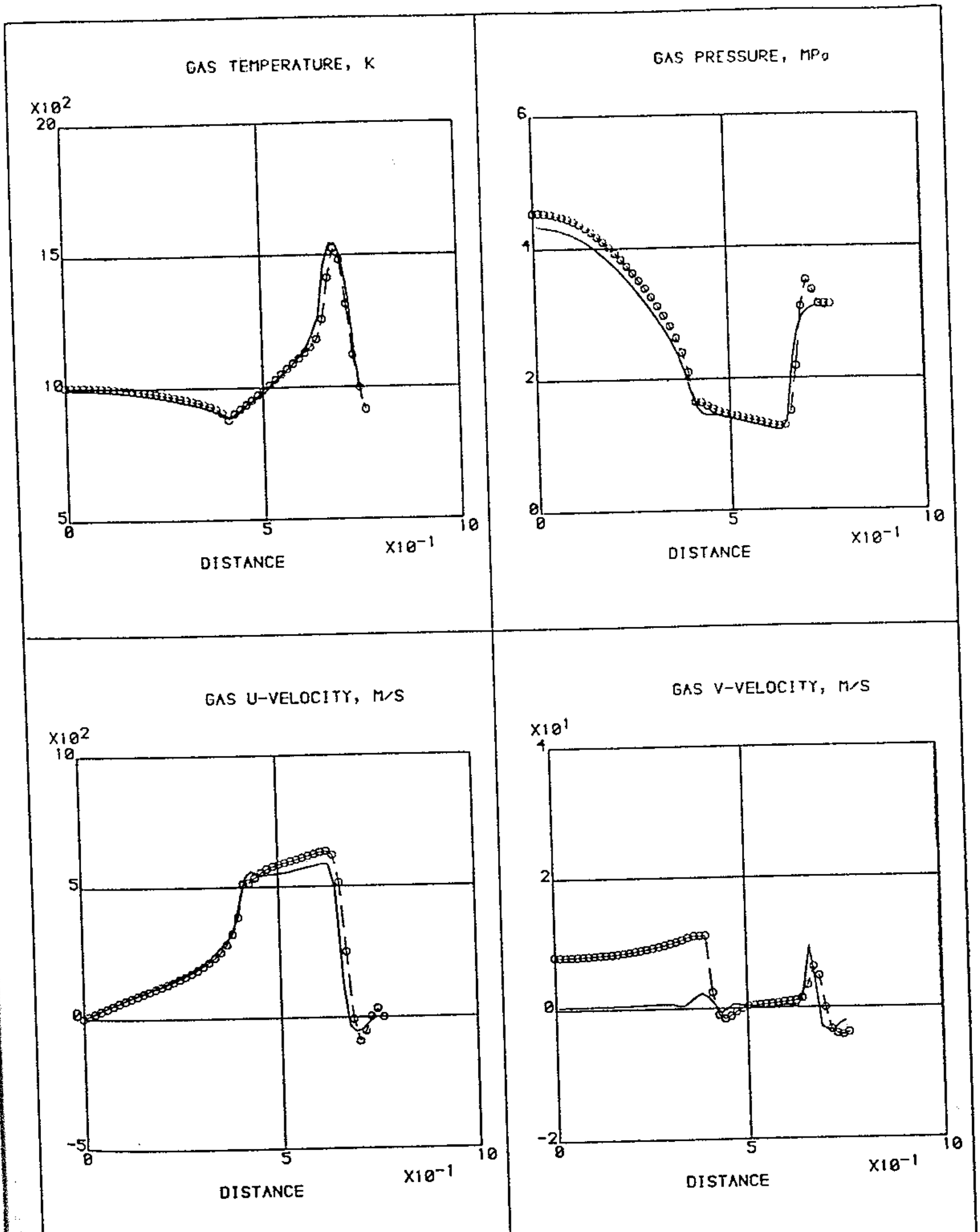


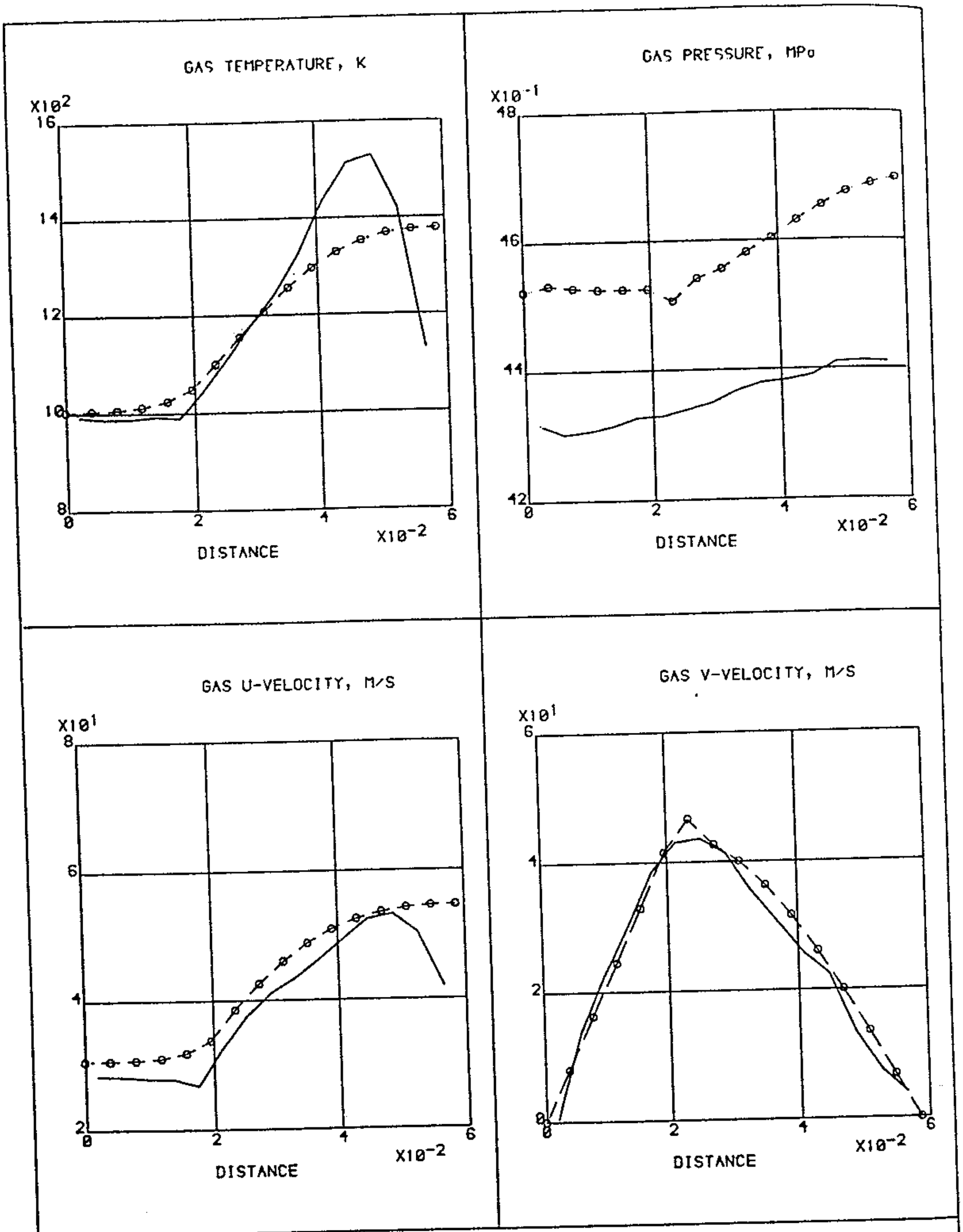

FIGURE 5 : TWO PHASE 2-D TEST PROBLEM. SOLUTION AT TIME = 0.75 MS




TWO-PHASE 2-D TEST PROBLEM. SOLUTION AT TIME = 0.75 MS.
FIGURE 6 :NUMERICAL SOLUTIONS USING WAFBC2 (FULL LINE)
AND ABC2 (SYMBOLS PLUS DASHED LINE) FOR J=1.

FIGURE 5 : TWO PHASE 2-D TEST PROBLEM. SOLUTION AT TIME = 0.75 MS

WAFBC2




TWO-PHASE 2-D TEST PROBLEM. SOLUTION AT TIME = 0.75 MS.
FIGURE 7 :NUMERICAL SOLUTIONS USING WAFBC2 (FULL LINE) AND ABC2 (SYMBOLS PLUS DASHED LINE) FOR I=4.

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INTRODUCTION

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